Further Mathematics SL

First examinations 2006

DIPLOMA PROGRAMME

FURTHER MATHEMATICS SL

First examinations 2006

International Baccalaureate Organization

Buenos Aires **Cardiff** **Geneva** **New York** **Singapore**

Diploma Programme
Further Mathematics SL

International Baccalaureate Organization, Geneva, CH-1218, Switzerland

First published in September 2004

by the International Baccalaureate Organization
Peterson House, Malthouse Avenue, Cardiff Gate
Cardiff, Wales GB CF23 8GL
UNITED KINGDOM
Tel: + 44 29 2054 7777
Fax: + 44 29 2054 7778
Web site: www.ibo.org

IBO merchandise and publications in its official and working languages can be purchased through the online catalogue at www.ibo.org, found by selecting *Publications* from the shortcuts box. General ordering queries should be directed to the sales department in Cardiff.
Tel: +44 29 2054 7746
Fax: +44 29 2054 7779
E-mail: sales@ibo.org

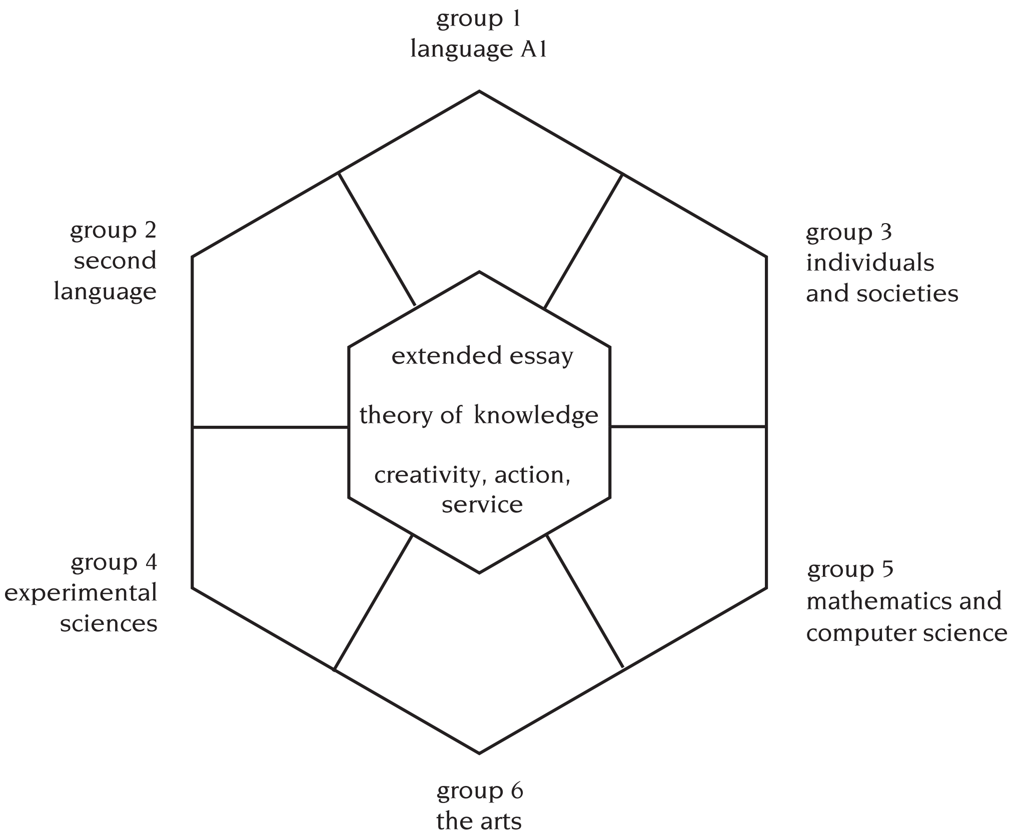
Printed in the United Kingdom by Antony Rowe Ltd, Chippenham, Wiltshire.

CONTENTS

INTRODUCTION

The International Baccalaureate Diploma Programme (DP) is a rigorous pre-university course of studies, leading to examinations, that meets the needs of highly motivated secondary school students between the ages of 16 and 19 years. Designed as a comprehensive two-year curriculum that allows its graduates to fulfill requirements of various national education systems, the DP model is based on the pattern of no single country but incorporates the best elements of many. The DP is available in English, French and Spanish.

The programme model is displayed in the shape of a hexagon with six academic areas surrounding the core. Subjects are studied concurrently and students are exposed to the two great traditions of learning: the humanities and the sciences.



DP students are required to select one subject from each of the six subject groups. At least three and not more than four are taken at higher level (HL), the others at standard level (SL). HL courses represent 240 teaching hours; SL courses cover 150 hours. By arranging work in this fashion, students are able to explore some subjects in depth and some more broadly over the two-year period; this is a deliberate compromise between the early specialization preferred in some national systems and the breadth found in others.

Distribution requirements ensure that the science-orientated student is challenged to learn a foreign language and that the natural linguist becomes familiar with science laboratory procedures. While overall balance is maintained, flexibility in choosing HL concentrations allows the student to pursue areas of personal interest and to meet special requirements for university entrance.

Successful DP students meet three requirements in addition to the six subjects. The interdisciplinary theory of knowledge (TOK) course is designed to develop a coherent approach to learning that transcends and unifies the academic areas and encourages appreciation of other cultural perspectives. The extended essay of some 4,000 words offers the opportunity to investigate a topic of special interest and acquaints students with the independent research and writing skills expected at university. Participation in the creativity, action, service (CAS) requirement encourages students to be involved in creative pursuits, physical activities and service projects in the local, national and international contexts.

First examinations 2006

NATURE OF THE SUBJECT

Introduction

The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives provides a clear and sufficient rationale for making the study of this subject compulsory within the DP.

Summary of courses available

Because individual students have different needs, interests and abilities, there are four different courses in mathematics. These courses are designed for different types of students: those who wish to study mathematics in depth, either as a subject in its own right or to pursue their interests in areas related to mathematics; those who wish to gain a degree of understanding and competence better to understand their approach to other subjects; and those who may not as yet be aware how mathematics may be relevant to their studies and in their daily lives. Each course is designed to meet the needs of a particular group of students. Therefore, great care should be taken to select the course that is most appropriate for an individual student.

In making this selection, individual students should be advised to take account of the following types of factor.

- Their own abilities in mathematics and the type of mathematics in which they can be successful
- Their own interest in mathematics, and those particular areas of the subject that may hold the most interest for them
- Their other choices of subjects within the framework of the DP
- Their academic plans, in particular the subjects they wish to study in future
- Their choice of career

Teachers are expected to assist with the selection process and to offer advice to students about how to choose the most appropriate course from the four mathematics courses available.

Mathematical studies SL

This course is available at SL only. It caters for students with varied backgrounds and abilities. More specifically, it is designed to build confidence and encourage an appreciation of mathematics in students who do not anticipate a need for mathematics in their future studies. Students taking this course need to be already equipped with fundamental skills and a rudimentary knowledge of basic processes.

Mathematics SL

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

Mathematics HL

This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems.

Further mathematics SL

This course is available at SL only. It caters for students with a good background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students will intend to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications.

Further mathematics SL—course details

This course caters for students with a good background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students will intend to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications.

The nature of the subject is such that it focuses on different branches of mathematics to encourage students to appreciate the diversity of the subject. Students should be equipped at this stage in their mathematical progress to begin to form an overview of the characteristics that are common to all mathematical thinking, independent of topic or branch.

All categories of student can register for mathematics HL only **or** for further mathematics SL only **or** for both. However, candidates registering for further mathematics SL will be presumed to know the topics in the core syllabus of mathematics HL and to have studied one of the options, irrespective of whether they have also registered for mathematics HL.

Examination questions are intended to be comparable in difficulty with those set on the four options in the mathematics HL course. The challenge for students will be to reach an equivalent level of understanding across these four topics.

AIMS

The aims of all courses in group 5 are to enable students to:

- appreciate the multicultural and historical perspectives of all group 5 courses
- enjoy the courses and develop an appreciation of the elegance, power and usefulness of the subjects
- develop logical, critical and creative thinking
- develop an understanding of the principles and nature of the subject
- employ and refine their powers of abstraction and generalization
- develop patience and persistence in problem solving
- appreciate the consequences arising from technological developments
- transfer skills to alternative situations and to future developments
- communicate clearly and confidently in a variety of contexts.

Internationalism

One of the aims of this course is to enable students to appreciate the multiplicity of cultural and historical perspectives of mathematics. This includes the international dimension of mathematics. Teachers can exploit opportunities to achieve this aim by discussing relevant issues as they arise and making reference to appropriate background information. For example, it may be appropriate to encourage students to discuss:

- differences in notation
- the lives of mathematicians set in a historical and/or social context
- the cultural context of mathematical discoveries
- the ways in which specific mathematical discoveries were made and the techniques used to make them
- how the attitudes of different societies towards specific areas of mathematics are demonstrated
- the universality of mathematics as a means of communication.

OBJECTIVES

Having followed any one of the mathematics courses in group 5, students are expected to know and use mathematical concepts and principles. In particular, students must be able to:

- read, interpret and solve a given problem using appropriate mathematical terms
- organize and present information and data in tabular, graphical and/or diagrammatic forms
- know and use appropriate notation and terminology
- formulate a mathematical argument and communicate it clearly
- select and use appropriate mathematical strategies and techniques
- demonstrate an understanding of both the significance and the reasonableness of results
- recognize patterns and structures in a variety of situations, and make generalizations
- recognize and demonstrate an understanding of the practical applications of mathematics
- use appropriate technological devices as mathematical tools
- demonstrate an understanding of and the appropriate use of mathematical modelling.

SYLLABUS OUTLINE

Further mathematics SL — Total 150 hrs

The course consists of the study of one geometry topic and the four mathematics HL option topics. Students must also be familiar with the topics listed as presumed knowledge and in the core syllabus for the mathematics HL course.

Geometry syllabus content — 30 hrs

Requirements

Students must study all the sub-topics in this topic, as listed in the syllabus details.

Topic 1—Geometry

Mathematics HL options syllabus content — 120 hrs

Requirements

Students must study all the sub-topics in all of the following topics as listed in the syllabus details.

Students will be presumed to have studied one of the option topics as part of the mathematics HL course. Consequently, this portion of the further mathematics SL course is regarded as having a total teaching time of 120 hours, not 160.

Topic 2—Statistics and probability	40 hrs
Topic 3—Sets, relations and groups	40 hrs
Topic 4—Series and differential equations	40 hrs
Topic 5—Discrete mathematics	40 hrs

SYLLABUS DETAILS

Format of the syllabus

The syllabus to be taught is presented as three columns.

- **Content**: the first column lists, under each topic, the sub-topics to be covered.
- **Amplifications/inclusions**: the second column contains more explicit information on specific sub-topics listed in the first column. This helps to define what is required in terms of preparing for the examination.
- **Exclusions**: the third column contains information about what is not required in terms of preparing for the examination.

Teaching notes linked to the syllabus content are contained in a separate publication.

Course of study

Teachers are required to teach all the sub-topics listed under the five topics.

The topics in the syllabus do not need to be taught in the order in which they appear in this guide. Teachers should therefore construct a course of study that is tailored to the needs of their students and that integrates the areas covered by the syllabus.

Time allocation

The recommended teaching time for a standard level subject is 150 hours. The time allocations given in this guide are approximate, and are intended to suggest how the hours allowed for teaching the syllabus might be allocated. However, the exact time spent on each topic depends on a number of factors, including the background knowledge and level of preparedness of each student. Teachers should therefore adjust these timings to correspond with the needs of their students.

Use of calculators

Students are expected to have access to a graphic display calculator (GDC) at all times during the course. The minimum requirements are reviewed as technology advances, and updated information will be provided to schools. It is expected that teachers and schools monitor calculator use with reference to the calculator policy. Regulations covering the types of calculator allowed are provided in the *Vade Mecum*.

There are specific requirements for calculators used by students studying the statistics and probability topic.

Mathematics HL, further mathematics SL information booklet

Because each student is required to have access to a clean copy of this booklet during the examination, it is recommended that teachers ensure students are familiar with the contents of this document from the beginning of the course. The booklet is provided by the IBO and is published separately.

Teacher support materials

A variety of teacher support materials relate to this guide. These materials will include specimen examination papers and markschemes and suggestions to help teachers integrate the use of GDCs into their teaching. These will be available to all schools.

External assessment guidelines

It is recommended that teachers familiarize themselves with the section on external assessment guidelines, as this contains important information about the examination papers. In particular, students need to be familiar with notation the IBO uses and the command terms, as these will be used without explanation in the examination papers.

Geometry syllabus content

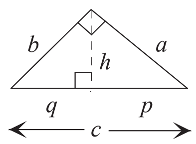
Note: proof forms a common thread throughout the five topics in the further mathematics course, including the techniques of: direct proof; indirect proof; both contrapositive and proof by contradiction; and induction.

Topic 1—Geometry 30 hrs

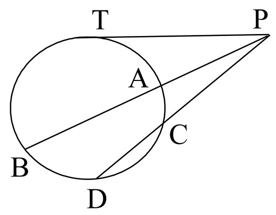
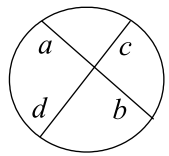
Aims

The aim of this section is to develop students' geometric intuition, visualization and deductive reasoning.

Details

	Content	Amplifications/inclusions	Exclusions
1.1	Triangles: medians; altitudes; angle bisectors; perpendicular bisectors of sides.	Euler's circle (the nine-point circle).	
	Concurrency: orthocentre; incentre; circumcentre; centroid, Euler line.	Proof of concurrency theorems.	
1.2	Euclid's theorem for proportional segments in a right-angled triangle.	Knowledge that the proportional segments p, q satisfy the following:  $h^2 = pq$ $a^2 = pc$ $b^2 = qc$.	
	Proportional division of a line segment (internal and external); the harmonic ratio; proportional segments in right-angled triangles.		

Topic 1—Geometry (continued)

	Content	Amplifications/inclusions	Exclusions
1.3	Circle geometry:	The equation of a circle.	
	tangents; arcs, chords and secants.	The tangent-secant and secant-secant theorems:  $PT^2 = PA \times PB = PC \times PD$. The intersecting chords theorem:  $ab = cd$.	On examination papers: questions that require constructions with ruler and compasses will not be set.
	The power of a point with respect to a circle.	See below.	
	Inscribed and circumscribed polygons; properties of cyclic quadrilaterals.		

Topic 1—Geometry (continued)

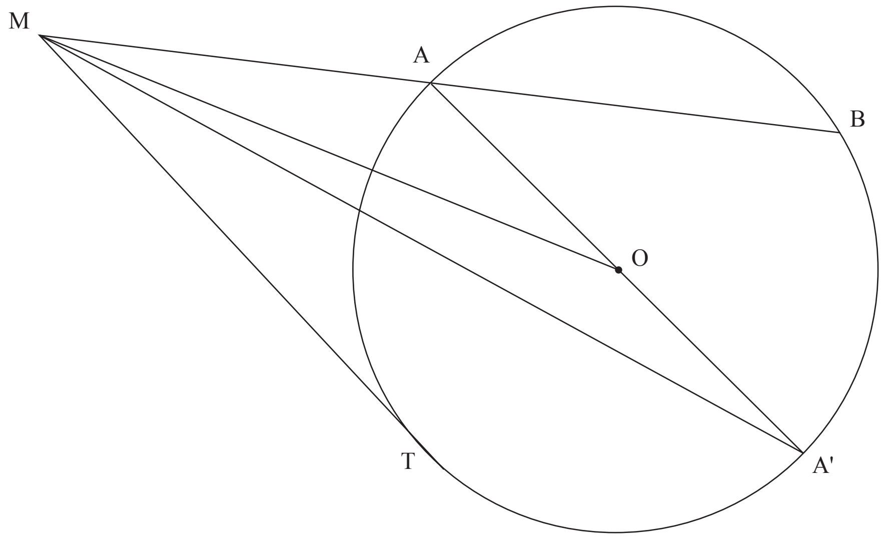
	Content	Amplifications/inclusions	Exclusions
1.4	Apollonius' circle theorem (circle of Apollonius); Stewart's theorem; Apollonius' theorem; Menelaus' theorem; Ceva's theorem; Simson's (Wallace's) line; Ptolemy's theorem; angle bisector theorem.	The use of these theorems to prove further results.	
	Proof of these theorems.		
	The concept of locus.	Loci of straight lines and of circles. On examination papers: locus questions may be set on sections 1.1 to 1.4, and on straight lines and circles.	Conic sections: parabolas; ellipses; hyperbolas.

Topic 1—Geometry (continued)

Power of a point with respect to a circle

There are a few equivalent definitions of the power of a point with respect to a circle. Here is a summary of the equivalent definitions.

Consider a circle C with centre O and radius r and a point M in the plane of the circle.



The following expressions are equivalent and are called "the power of M with respect to the circle".

1. The scalar product $\vec{\mathrm{MA}} \cdot \vec{\mathrm{MA'}}$ where A and A′ are diametrically opposite.
2. $\vec{\mathrm{MA}} \cdot \vec{\mathrm{MB}}$.
3. $d^2 - r^2$, where $d = \mathrm{MO}$.
4. MT^2 where MT is a tangent to the circle through the given point M.
5. $(x_0 - h)^2 + (y_0 - k)^2 - r^2$, where (x_0, y_0) are the Cartesian coordinates of the point M, and (h, k) are the coordinates of the centre O of the circle.

From this definition it is clear that the power of M with respect to C is positive when the point is outside the circle, negative when it is inside the circle, and zero when it is on the circle. (The proof of the equivalence of these expressions is a good activity for students to do).

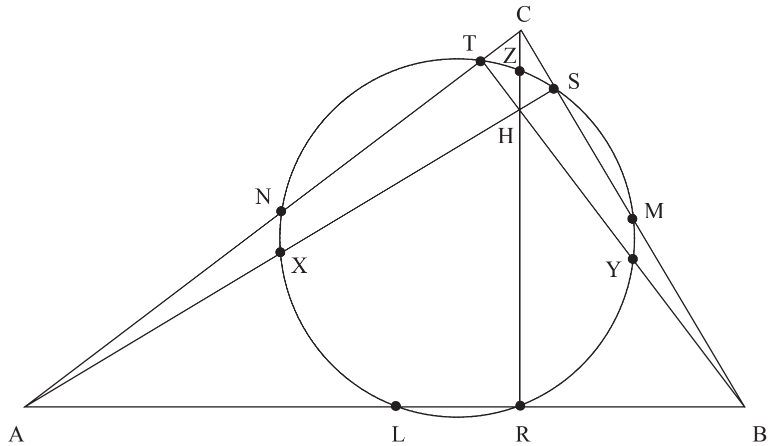
Geometry theorems

Teachers and students should be aware that some of the theorems mentioned in this section may be known by other names or some names of theorems may be associated with different statements in some textbooks. To avoid confusion, on examination papers, theorems that may be misinterpreted are defined below.

Euler theorem (nine-point circle theorem)

Given any triangle ABC, let H be the intersection of the three altitudes. There is a circle that passes through these nine special points:

- the midpoints L, M, N of the three sides
- the points R, S, T, where the three altitudes of the triangle meet the sides
- the midpoints X, Y, Z, of [HA], [HB], [HC].



Topic 1—Geometry (continued)

Apollonius' circle theorem (circle of Apollonius)

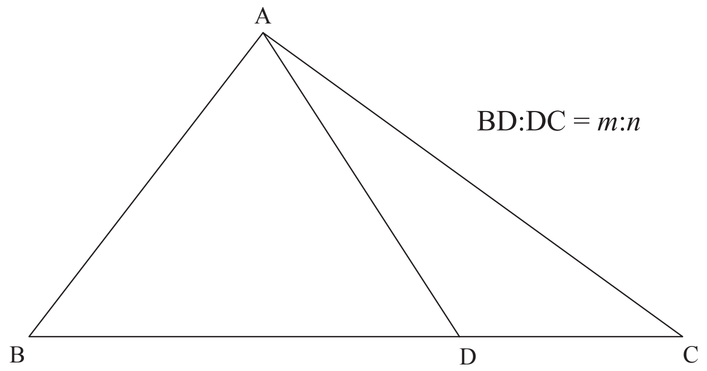
If A, B are two fixed points such that $\frac{\mathrm{PA}}{\mathrm{PB}}$ is a constant not equal to one then the locus of P is a circle.

This is called the circle of Apollonius.

Included: the converse of this theorem.

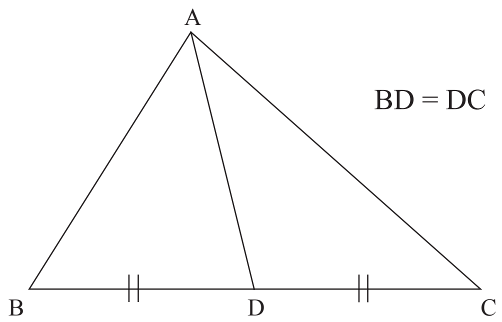
Stewart's theorem

If D is any point on the base [BC] of a triangle ABC, dividing BC in the ratio $m:n$, then $n\mathrm{AB}^2 + m\mathrm{AC}^2 = (m+n)\mathrm{AD}^2 + m\mathrm{CD}^2 + n\mathrm{BD}^2$.



Apollonius' theorem (special case of Stewart's theorem, with $m = n$)

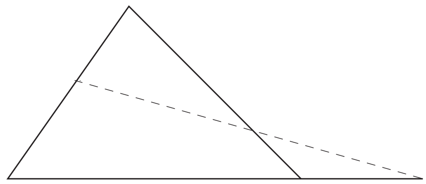
If D is the midpoint of the base [BC] of a triangle ABC, then $\mathrm{AB}^2 + \mathrm{AC}^2 = 2(\mathrm{AD}^2 + \mathrm{BD}^2)$.



Menelaus' theorem

If a transversal meets the sides [BC], [CA], [AB] of a triangle at D, E, F respectively, then

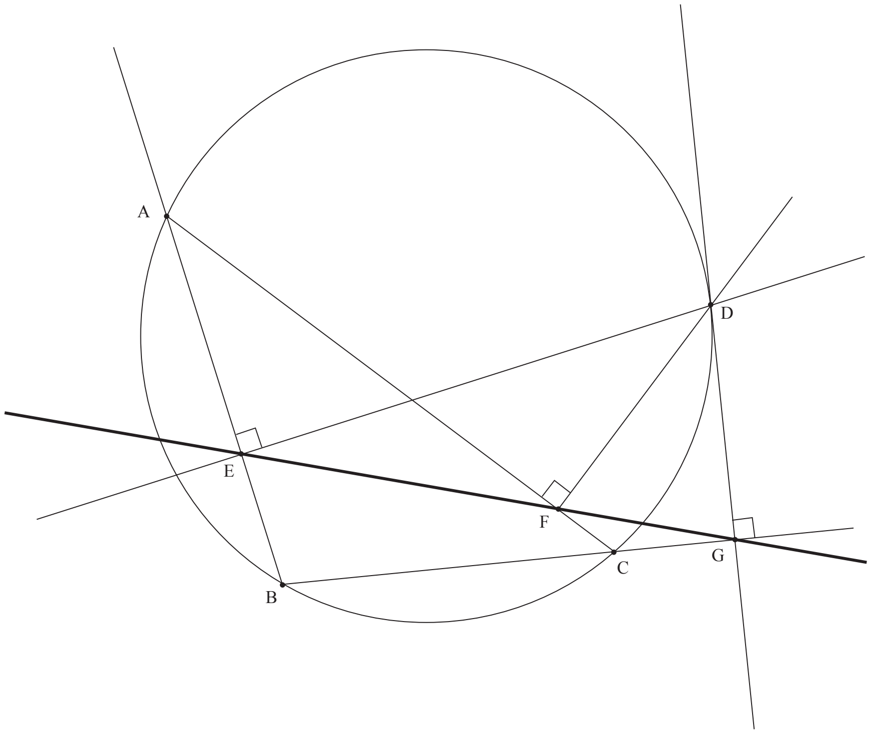
$$\frac{\mathrm{BD}}{\mathrm{DC}} \times \frac{\mathrm{CE}}{\mathrm{EA}} \times \frac{\mathrm{AF}}{\mathrm{FB}} = -1 .$$



Topic 1—Geometry (continued)

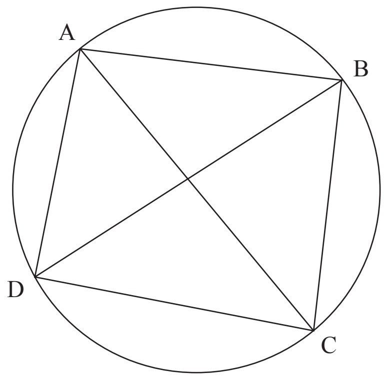
Simson's line (Wallace's line)

Let D be any point on the circumscribed circle of the triangle ABC. The feet of the perpendiculars from D to each line segment $[AB]$, $[AC]$ and $[BC]$ are collinear. The line (EFG) is called Simson's line (or Wallace's line).



Ptolemy's theorem

If a quadrilateral is cyclic, the sum of the products of the two pairs of opposite sides equals the products of the diagonals. That is, for a cyclic quadrilateral ABCD, $AB \times CD + BC \times DA = AC \times BD$.

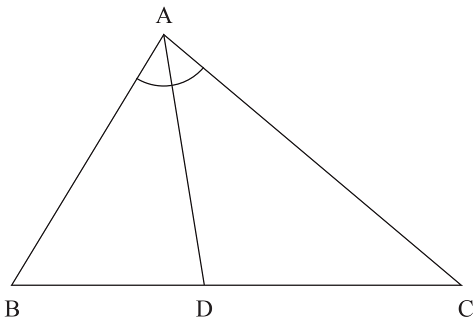


Topic 1—Geometry (continued)

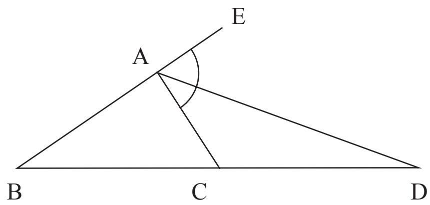
Angle bisector theorem

The angle bisector of an angle of a triangle divides the side of the triangle opposite the angle into segments proportional to the sides adjacent to the angle.

If ABC is the given triangle with (AD) as the bisector of angle BAC intersecting (BC) at point D, then $\frac{\mathrm{BD}}{\mathrm{DC}} = \frac{\mathrm{AB}}{\mathrm{AC}}$.



$\mathrm{B\hat{A}D} = \mathrm{C\hat{A}D}$



$\mathrm{C\hat{A}D} = \mathrm{E\hat{A}D}$

Included: the converse of this theorem.

Mathematics HL options syllabus content

Topic 2—Statistics and probability 40 hrs

Aims

The aims of this option are to allow students the opportunity to approach statistics in a practical way; to demonstrate a good level of statistical understanding; and to understand which situations apply and to interpret the given results. It is expected that GDCs will be used throughout this option and that the minimum requirement of a GDC will be to find pdf, cdf, inverse cdf, *p*-values and test statistics including calculations for the following distributions: binomial, Poisson, normal, *t* and chi-squared. Students are expected to set up the problem mathematically and then read the answers from the GDC, indicating this within their written answers. Calculator-specific or brand-specific language should not be used within these explanations.

Details

	Content	Amplifications/inclusions	Exclusions
2.1	Expectation algebra.	$\mathrm{E}(aX+b)=a\mathrm{E}(X)+b$; $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$.	
	Linear transformation of a single random variable.		
	Mean and variance of linear combinations of two independent random variables.	$\mathrm{E}(a_1X_1 \pm a_2X_2)=a_1\mathrm{E}(X_1)\pm a_2\mathrm{E}(X_2)$; $\mathrm{Var}(a_1X_1 \pm a_2X_2)=a_1^2\mathrm{Var}(X_1)+a_2^2\mathrm{Var}(X_2)$.	
	Extension to linear combinations of *n* independent random variables.		

Topic 2—Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
2.2	Cumulative distribution functions.		Formal treatment of proof of means and variances.
	Discrete distributions: uniform, Bernoulli, binomial, negative binomial, Poisson, geometric, hypergeometric.	Probability mass functions, means and variances.	
	Continuous distributions: uniform, exponential, normal.	Probability density functions, means and variances.	
2.3	Distribution of the sample mean.		Sampling without replacement.
	The distribution of linear combinations of independent normal random variables. In particular $X \sim \mathrm{N}(\mu, \sigma^2) \Rightarrow \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$.	A linear combination of independent normally distributed random variables is also normally distributed.	
	The central limit theorem.		Proof of the central limit theorem.
	The approximate normality of the proportion of successes in a large sample.	The extension of these results for large samples to distributions that are not normal, using the central limit theorem.	Distributions that do not satisfy the central limit theorem.

Topic 2—Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
2.4	Finding confidence intervals for the mean of a population. Finding confidence intervals for the proportion of successes in a population.	Use of the normal distribution when σ is known and the *t*-distribution when σ is unknown (regardless of sample size). The case of paired samples (matched pairs) could be tested as an example of a single sample technique.	The difference of means and the difference of proportions.
2.5	Significance testing for a mean. Significance testing for a proportion. Null and alternative hypotheses H_0 and H_1. Type I and Type II errors. Significance levels; critical region, critical values, *p*-values; one-tailed and two-tailed tests.	Use of the normal distribution when σ is known and the *t*-distribution when σ is unknown. The case of paired samples (matched pairs) could be tested as an example of a single sample technique.	The difference of means and the difference of proportions.

Topic 2—Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
2.6	The chi-squared distribution: degrees of freedom, ν .		
	The χ^2 statistic, $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$.	Awareness of the fact that χ^2_{calc} is a measure of the discrepancy between observed and expected values.	
	The χ^2 goodness of fit test.	Test for goodness of fit for all of the above distributions; the requirement to combine classes with expected frequencies of less than 5.	
	Contingency tables: the χ^2 test for the independence of two variables.		Yates' continuity correction for $\nu = 1$.

Topic 3—Sets, relations and groups 40 hrs

Aims

The aims of this option are to provide the opportunity to study some important mathematical concepts, and introduce the principles of proof through abstract algebra.

Details

	Content	Amplifications/inclusions	Exclusions
3.1	Finite and infinite sets. Subsets. Operations on sets: union; intersection; complement, set difference, symmetric difference.		
	De Morgan's laws; distributive, associative and commutative laws (for union and intersection).	Illustration of these laws using Venn diagrams.	Proofs of these laws.
3.2	Ordered pairs: the Cartesian product of two sets.		
	Relations; equivalence relations; equivalence classes.	An equivalence relation on a set induces a partition of the set.	
3.3	Functions: injections; surjections; bijections.	The term "codomain".	
	Composition of functions and inverse functions.	Knowledge that the function composition is not a commutative operation and that if f is a bijection from set A onto set B then f^{-1} exists and is a bijection from set B onto set A.	

Topic 3—Sets, relations and groups (continued)

	Content	Amplifications/inclusions	Exclusions
3.4	Binary operations.	A binary operation $*$ on a non-empty set S is a rule for combining any two elements $a, b \in S$ to give a unique element c. That is, in this definition, a binary operation is not necessarily closed. On examination papers: candidates may be required to test whether a given operation satisfies the closure condition.	
	Operation tables (Cayley tables).	Operation tables with the Latin square property (every element appears once only in each row and each column).	
3.5	Binary operations with associative, distributive and commutative properties.	The arithmetic operations in $\mathbb{R}$ and $\mathbb{C}$; matrix operations.	
3.6	The identity element e.	Both the right-identity $a * e = a$ and left-identity $e * a = a$ must hold if e is an identity element.	
	The inverse a^{-1} of an element a.	Both $a * a^{-1} = e$ and $a^{-1} * a = e$ must hold.	
	Proof that left-cancellation and right-cancellation by an element a hold, provided that a has an inverse.		
	Proofs of the uniqueness of the identity and inverse elements.		

Topic 3—Sets, relations and groups (continued)

	Content	Amplifications/inclusions	Exclusions
3.7	The axioms of a group $\{G,*\}$.	For the set G under a given operation $*$: • G is closed under $*$ • $*$ is associative • G contains an identity element • each element in G has an inverse in G.	
	Abelian groups.	$a*b=b*a$, for all $a,b\in G$.	
3.8	The groups: • $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$ and $\mathbb{C}$ under addition • matrices of the same order under addition • 2×2 invertible matrices under multiplication • integers under addition modulo n		
	• groups of transformations	The composition T_1T_2 denotes T_2 followed by T_1.	
	• symmetries of an equilateral triangle, rectangle and square • invertible functions under composition of functions		
	• permutations under composition of permutations.	On examination papers: the form $p=\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$ will be used to represent the mapping $1\to 3$, $2\to 1$, $3\to 2$.	

Topic 3—Sets, relations and groups (continued)

	Content	Amplifications/inclusions	Exclusions
3.9	Finite and infinite groups.	Latin square property of a group table.	
	The order of a group element and the order of a group.		
3.10	Cyclic groups.	Generators.	
	Proof that all cyclic groups are Abelian.		
3.11	Subgroups, proper subgroups.		
	Use and proof of subgroup tests.	Suppose G is a group and H is a non-empty subset of G. H is a subgroup of G if $ab^{-1} \in H$ whenever $a, b \in H$.	
		Suppose G is a finite group and H is a non-empty subset of G. H is a subgroup of G if H is closed under the group operation.	
	Lagrange's theorem.		On examination papers: questions requiring the proof of Lagrange's theorem will not be set.
	Use and proof of the result that the order of a finite group is divisible by the order of any element. (Corollary to Lagrange's theorem.)		

Topic 3—Sets, relations and groups (continued)

	Content	Amplifications/inclusions	Exclusions
3.12	Isomorphism of groups.	Infinite groups as well as finite groups. Two groups $\{G, \circ\}$ and $\{H, \bullet\}$ are isomorphic if there exists a bijection $f: G \to H$ such that $f(a \circ b) = f(a) \bullet f(b)$ for all $a, b \in G$. The function $f: G \to H$ is an isomorphism.	
	Proof of isomorphism properties for identities and inverses.	Identity: let e_1 and e_2 be the identity elements of G, H respectively, then $f(e_1) = e_2$. Inverse: $f(a^{-1}) = \left(f(a)\right)^{-1}$ for all $a \in G$.	

Topic 4—Series and differential equations 40 hrs

Aims

The aims of this option are to introduce limit theorems and convergence of series, and to use calculus results to solve differential equations.

Details

	Content	Amplifications/inclusions	Exclusions
4.1	Infinite sequences of real numbers.		
	Limit theorems as n approaches infinity.	Limit of sum, difference, product, quotient; squeeze theorem.	
	Limit of a sequence.	Formal definition: the sequence $\{u_n\}$ converges to the limit L, if for any $\varepsilon > 0$, there is a positive integer N such that $\lvert u_n - L \rvert < \varepsilon$, for all $n > N$.	
	Improper integrals of the type $\int_a^{\infty} f(x)\,\mathrm{d}x$.		
	The integral as a limit of a sum; lower sum and upper sum.		

Topic 4—Series and differential equations (continued)

	Content	Amplifications/inclusions	Exclusions
4.2	Convergence of infinite series.	The sum of a series is the limit of the sequence of its partial sums.	
	Partial fractions and telescoping series (method of differences).	Simple linear non-repeated denominators.	
	Tests for convergence: comparison test; limit comparison test; ratio test; integral test.	Students should be aware that if $\lim_{x\to\infty} x_n = 0$ then the series is not necessarily convergent, but if $\lim_{x\to\infty} x_n \neq 0$, the series diverges.	
	The *p*-series, $\sum \frac{1}{n^p}$.	$\sum \frac{1}{n^p}$ is convergent for $p > 1$ and divergent otherwise. When $p = 1$, this is the harmonic series.	
	Use of integrals to estimate sums of series.		
4.3	Series that converge absolutely.		
	Series that converge conditionally.		
	Alternating series.	Conditions for convergence. The absolute value of the truncation error is less than the next term in the series.	
4.4	Power series: radius of convergence and interval of convergence. Determination of the radius of convergence by the ratio test.		

Topic 4—Series and differential equations (continued)

	Content	Amplifications/inclusions	Exclusions
4.5	Taylor polynomials and series, including the error term.	Applications to the approximation of functions; formulae for the error term, both in terms of the value of the $(n+1)^{\text{th}}$ derivative at an intermediate point, and in terms of an integral of the $(n+1)^{\text{th}}$ derivative.	Proof of Taylor's theorem.
		Differentiation and integration of series (valid only on the interval of convergence of the initial series).	Use of products and quotients to obtain other series.
	Maclaurin series for e^x, $\sin x$, $\cos x$, $\arctan x$, $\ln(1+x)$, $(1+x)^p$. Use of substitution to obtain other series.	Intervals of convergence for these Maclaurin series. Example: e^{x^2}.	
	The evaluation of limits of the form $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ using l'Hôpital's Rule and/or the Taylor series.	Cases where the derivatives of $f(x)$ and $g(x)$ vanish for $x = a$.	Proof of l'Hôpital's Rule.

Topic 4—Series and differential equations (continued)

	Content	Amplifications/inclusions	Exclusions
4.6	First order differential equations: geometric interpretation using slope fields; numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$ using Euler's method. Homogeneous differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = f\left(\frac{y}{x}\right)$ using the substitution $y = vx$. Solution of $y' + P(x)y = Q(x)$, using the integrating factor.	 $y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$, where h is a constant.	

Topic 5—Discrete mathematics 40 hrs

Aims

The aim of this option is to provide the opportunity for students to engage in logical reasoning, algorithmic thinking and applications.

Details

	Content	Amplifications/inclusions	Exclusions
5.1	Division and Euclidean algorithms.	The theorem $a \mid b$ and $a \mid c \Rightarrow a \mid (bx \pm cy)$ where $x, y \in \mathbb{Z}$. The division algorithm $a = bq + r$, $0 \le r < b$.	
	The greatest common divisor, $\gcd(a,b)$, and the least common multiple, $\operatorname{lcm}(a,b)$, of integers a and b.	The Euclidean algorithm for determining the greatest common divisor of two integers.	
	Relatively prime numbers; prime numbers and the fundamental theorem of arithmetic.		Proof of the fundamental theorem of arithmetic.
5.2	Representation of integers in different bases.	On examination papers: questions that go beyond base 16 are unlikely to be set.	
5.3	Linear diophantine equations $ax + by = c$.	General solutions required and solutions subject to constraints. For example, all solutions must be positive.	
5.4	Modular arithmetic. Linear congruences. Chinese remainder theorem.		
5.5	Fermat's little theorem.	$a^p \equiv a (\operatorname{mod} p)$ where p is prime.	On examination papers: questions requiring proof of the theorem will not be set.

Topic 5—Discrete mathematics (continued)

	Content	Amplifications/inclusions	Exclusions
5.6	Graphs, vertices, edges. Adjacent vertices, adjacent edges.	Two vertices are adjacent if they are joined by an edge. Two edges are adjacent if they have a common vertex.	
	Simple graphs; connected graphs; complete graphs; bipartite graphs; planar graphs, trees, weighted graphs.	Euler's relation: $v - e + f = 2$; theorems for planar graphs including $e \le 3v - 6$, $e \le 2v - 4$, κ_5 and $\kappa_{3,3}$ are not planar.	
	Subgraphs; complements of graphs.		
	Graph isomorphism.	Simple graphs only for isomorphism.	
5.7	Walks, trails, paths, circuits, cycles.	A connected graph contains a Eulerian circuit if and only if every vertex of the graph is of even degree.	Dirac's theorem for Hamiltonian cycles.
	Hamiltonian paths and cycles; Eulerian trails and circuits.		
5.8	Adjacency matrix.	Applications to isomorphism and of the powers of the adjacency matrix to number of walks.	
	Cost adjacency matrix.		
5.9	Graph algorithms: Prim's; Kruskal's; Dijkstra's.	These are examples of "greedy" algorithms.	

Topic 5—Discrete mathematics (continued)

	Content	Amplifications/inclusions	Exclusions
5.10	"Chinese postman" problem ("route inspection").	To determine the shortest route around a weighted graph going along each edge at least once (route inspection algorithm).	Graphs with more than two vertices of odd degree.
	"Travelling salesman" problem.	To determine the Hamiltonian cycle of least weight in a weighted complete graph.	
	Algorithms for determining upper and lower bounds of the travelling salesman problem.		Graphs in which the triangle inequality is not satisfied.

Glossary of terminology for the discrete mathematics option

Introduction

Teachers and students should be aware that many different terminologies exist in graph theory and that different textbooks may employ different combinations of these. Examples of these are: vertex/node/junction/point; edge/route/arc; degree of a vertex/order; multiple edges/parallel edges; loop/self-loop.

In IBO examination questions, the terminology used will be as it appears in the syllabus. For clarity these terms are defined below.

Terminology

Graph	Consists of a set of vertices and a set of edges; an edge joins its endpoints (vertices)
Subgraph	A graph within a graph
Weighted graph	A graph in which each edge is allocated a number or weight
Loop	An edge whose endpoints are joined to the same vertex
Multiple edges	Occur if more than one edge joins the same pair of vertices
Walk	A sequence of linked edges
Trail	A walk in which no edge appears more than once
Path	A walk with no repeated vertices
Circuit	A walk that begins and ends at the same vertex, and has no repeated edges
Cycle	A walk that begins and ends at the same vertex, and has no other repeated vertices
Hamiltonian path	A path that contains all the vertices of the graph
Hamiltonian cycle	A cycle that contains all the vertices of the graph
Eulerian trail	A trail that contains every edge of a graph
Eulerian circuit	A circuit that contains every edge of a graph
Degree of a vertex	The number of edges joined to the vertex; a loop contributes two, one for each of its endpoints
Simple graph	A graph without loops or multiple edges
Complete graph	A simple graph where every vertex is joined to every other vertex
Connected graph	A graph that has a path joining every pair of vertices
Disconnected graph	A graph that has at least one pair of vertices not joined by a path

Tree	A connected graph that contains no cycles
Weighted tree	A tree in which each edge is allocated a number or weight
Spanning tree of a graph	A subgraph containing every vertex of the graph, which is also a tree
Minimum spanning tree	A spanning tree of a weighted graph that has the minimum total weight
Complement of a graph G	A graph with the same vertices as G but which has an edge between any two vertices if and only if G does not
Graph isomorphism between two simple graphs G and H	A one-to-one correspondence between vertices of G and H such that a pair of vertices in G is adjacent if and only if the corresponding pair in H is adjacent
Planar graph	A graph that can be drawn in the plane without any edge crossing another
Bipartite graph	A graph whose vertices can be divided into two sets and in which edges always join a vertex from one set to a vertex from the other set
Complete bipartite graph	A bipartite graph in which every vertex in one set is joined to every vertex in the other set
Adjacency matrix of G, denoted by $\boldsymbol{A}_G$	The adjacency matrix, $\boldsymbol{A}_G$, of a graph G with n vertices, is the $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j. Hence, the adjacency matrix will be symmetric about the diagonal.
Cost adjacency matrix of G, denoted by $\boldsymbol{C}_G$	The cost adjacency matrix, $\boldsymbol{C}_G$, of a graph G with n vertices is the $n \times n$ matrix in which the entry in row i and column j is the weight of the edges joining the vertices i and j.

ASSESSMENT OUTLINE

First examinations 2006

Further mathematics SL

External assessment 3 hrs 100%

Written papers

Paper 1 1 hr 35%

Four to six compulsory short-response questions based on the whole syllabus.

Paper 2 2 hrs 65%

Four to six compulsory extended-response questions based on the whole syllabus.

ASSESSMENT DETAILS

External assessment details 3 hrs 100%

General

Paper 1 and paper 2

These papers are externally set and externally marked. Together they contribute 100% of the final mark. These papers are designed to allow students to demonstrate what they know and what they can do. It is not intended that equal weight will be given to each of the five topics in the syllabus on one paper. The two papers between them will provide the syllabus coverage, but not all topics are necessarily assessed in every examination session.

Calculators

For both examination papers, students must have access to a GDC. Regulations covering the types of calculators allowed are provided in the *Vade Mecum*.

Mathematics HL, further mathematics SL information booklet

Each student must have access to a clean copy of the information booklet during the examination. One copy of this booklet is provided by the IBO as part of the examination papers mailing.

Awarding of marks

Marks may be awarded for method, accuracy, answers and reasoning, including interpretation.

In both papers, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example, diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

Paper 1 1 hr 35%

This paper consists of four to six compulsory short-response questions based on the whole syllabus.

Syllabus coverage

- Knowledge of **all** topics from the syllabus is required for this paper. However, not all topics are necessarily assessed in every examination session.
- The intention of this paper is to test students' knowledge across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A relatively small number of steps, fewer than those required for paper 2 questions, will be needed to solve paper 1 questions.
- Questions may be presented in the form of words, symbols, tables or diagrams, or combinations of these.

Mark allocation

- This paper is worth **60** marks, representing **35%** of the final mark.
- Questions may be unequal in terms of length and level of difficulty. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of each question.

Paper 2 — 2 hrs — 65%

This paper consists of four to six compulsory extended-response questions based on the whole syllabus. Students should spend up to 20 minutes in thought and reflection.

Syllabus coverage

- Knowledge of **all** topics from the syllabus is required for this paper. However, not all topics are necessarily assessed in every examination session.
- Individual questions may require knowledge of more than one topic from the syllabus.
- The intention of this paper is to test students' knowledge of the syllabus in depth. It should not be assumed that the separate topics will be given equal emphasis.

Question type

- Questions will require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem solving.

Mark allocation

- This paper is worth **120** marks, representing **65%** of the final mark.
- Questions may be unequal in terms of length and level of difficulty. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of each question.

Guidelines

Notation

Of the various notations in use, the IBO has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IBO notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

In a small number of cases, students may need to use alternative forms of notation in their written answers. This is because not all forms of IBO notation can be directly transferred into handwritten form. For vectors in particular the IBO notation uses a bold, italic typeface that cannot adequately be transferred into handwritten form. In this case, teachers should advise candidates to use alternative forms of notation in their written work (for example, $\vec{x}$, $\bar{x}$ or $\underset{\sim}{x}$).

Students must always use correct mathematical notation, not calculator notation.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\mathbb{C}$	the set of complex numbers, $\{a + \mathrm{i}b \mid a, b \in \mathbb{R}\}$
i	$\sqrt{-1}$
z	a complex number
z^*	the complex conjugate of z
$\lvert z \rvert$	the modulus of z
$\arg z$	the argument of z
$\mathrm{Re}\, z$	the real part of z

$\mathrm{Im}\, z$	the imaginary part of z
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\{x \mid \quad\}$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	the empty (null) set
U	the universal set
$\cup$	union
$\cap$	intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of
A'	the complement of the set A
$A \times B$	the Cartesian product of sets A and B (that is, $A \times B = \{(a, b) \mid a \in A,\ b \in B\}$)
$a \mid b$	a divides b
$a^{1/n}$, $\sqrt[n]{a}$	a to the power of $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$a^{1/2}$, $\sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geq 0$ then $\sqrt{a} \geq 0$)
$\lvert x \rvert$	the modulus or absolute value of x, that is $\begin{cases} x \text{ for } x \geq 0,\ x \in \mathbb{R} \\ -x \text{ for } x < 0,\ x \in \mathbb{R} \end{cases}$
$\equiv$	identity
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to

$\ngtr$	is not greater than
$\nless$	is not less than
$[a, b]$	the closed interval $a \le x \le b$
$]a, b[$	the open interval $a < x < b$
u_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + ... + u_n$
S_∞	the sum to infinity of a sequence, $u_1 + u_2 + ...$
$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + ... + u_n$
$\prod_{i=1}^{n} u_i$	$u_1 \times u_2 \times ... \times u_n$
$\binom{n}{r}$	$\frac{n!}{r!(n-r)!}$
$f : A \to B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	f is a function under which x is mapped to y
$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f
$f \circ g$	the composite function of f and g
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	the second derivative of y with respect to x

$f''(x)$	the second derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the n^{th} derivative of y with respect to x
$f^{(n)}(x)$	the n^{th} derivative of $f(x)$ with respect to x
$\int y\,\mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x between the limits $x=a$ and $x=b$
e^x	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	the natural logarithm of x, $\log_{\mathrm{e}} x$
sin, cos, tan	the circular functions
arcsin, arccos, arctan	the inverse circular functions
csc, sec, cot	the reciprocal circular functions
$\mathrm{A}(x, y)$	the point A in the plane with Cartesian coordinates x and y
$[\mathrm{AB}]$	the line segment with end points A and B
AB	the length of $[\mathrm{AB}]$
(AB)	the line containing points A and B
$\hat{A}$	the angle at A
$\mathrm{C\hat{A}B}$	the angle between $[\mathrm{CA}]$ and $[\mathrm{AB}]$
$\Delta\mathrm{ABC}$	the triangle whose vertices are A, B and C
$\boldsymbol{v}$	the vector $\boldsymbol{v}$
$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment from A to B
$\boldsymbol{a}$	the position vector $\overrightarrow{\mathrm{OA}}$
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	unit vectors in the directions of the Cartesian coordinate axes

$\lvert \boldsymbol{a} \rvert$	the magnitude of $\boldsymbol{a}$
$\lvert \overrightarrow{\mathrm{AB}} \rvert$	the magnitude of $\overrightarrow{\mathrm{AB}}$
$\boldsymbol{v} \cdot \boldsymbol{w}$	the scalar product of $\boldsymbol{v}$ and $\boldsymbol{w}$
$\boldsymbol{v} \times \boldsymbol{w}$	the vector product of $\boldsymbol{v}$ and $\boldsymbol{w}$
$\boldsymbol{A}^{-1}$	the inverse of the non-singular matrix $\boldsymbol{A}$
$\boldsymbol{A}^{\mathrm{T}}$	the transpose of the matrix $\boldsymbol{A}$
$\det \boldsymbol{A}$	the determinant of the square matrix $\boldsymbol{A}$
$\boldsymbol{I}$	the identity matrix
$\mathrm{P}(A)$	probability of event A
$\mathrm{P}(A')$	probability of the event "not A"
$\mathrm{P}(A \mid B)$	probability of the event A given B
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
P_x	probability distribution function $\mathrm{P}(X = x)$ of the discrete random variable X
$f(x)$	probability density function of the continuous random variable X
$F(x)$	cumulative distribution function of the continuous random variable X
$\mathrm{E}(X)$	the expected value of the random variable X
$\mathrm{Var}(X)$	the variance of the random variable X
μ	population mean
σ^2	population variance, $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$, where $n = \sum_{i=1}^{k} f_i$
σ	population standard deviation
$\bar{x}$	sample mean

s_n^2	sample variance, $s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{n}$, where $n = \sum_{i=1}^{k} f_i$
s_n	standard deviation of the sample
s_{n-1}^2	unbiased estimate of the population variance, $s_{n-1}^2 = \dfrac{n}{n-1} s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{n-1}$, where $n = \sum_{i=1}^{k} f_i$
$\mathrm{B}(n,\ p)$	binomial distribution with parameters *n* and *p*
$\mathrm{Po}(m)$	Poisson distribution with mean *m*
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
$X \sim \mathrm{B}(n, p)$	the random variable *X* has a binomial distribution with parameters *n* and *p*
$X \sim \mathrm{Po}(m)$	the random variable *X* has a Poisson distribution with mean *m*
$X \sim \mathrm{N}(\mu, \sigma^2)$	the random variable *X* has a normal distribution with mean μ and variance σ^2
Φ	cumulative distribution function of the standardized normal variable with distribution $\mathrm{N}(0, 1)$
ν	number of degrees of freedom
χ^2	chi-squared distribution
χ^2_{calc}	the chi-squared test statistic, where $\chi^2_{calc} = \sum \dfrac{(f_o - f_e)^2}{f_e}$
$A \setminus B$	the difference of the sets *A* and *B* (that is, $A \setminus B = A \cap B' = \{x \mid x \in A \text{ and } x \notin B\}$)
$A \Delta B$	the symmetric difference of the sets *A* and *B* (that is, $A \Delta B = (A \setminus B) \cup (B \setminus A)$)
κ_n	a complete graph with *n* vertices
$\kappa_{n,m}$	a complete bipartite graph with one set of *n* vertices and another set of *m* vertices
$\mathbb{Z}_p$	the set of equivalence classes $\{0, 1, 2, \ldots, p-1\}$ of integers modulo *p*

$\gcd(a,b)$	the greatest common divisor of integers a and b
$\operatorname{lcm}(a,b)$	the least common multiple of integers a and b
$\boldsymbol{A}_G$	the adjacency matrix of graph G
$\boldsymbol{C}_G$	the cost adjacency matrix of graph G

Glossary of command terms

The following command terms are used without explanation on examination papers. Teachers should familiarize themselves and their students with the terms and their meanings. This list is not exhaustive. Other command terms may be used, but it should be assumed that they have their usual meaning (for example, "explain" and "estimate"). The terms included here are those that sometimes have a meaning in mathematics that is different from the usual meaning.

Further clarification and examples can be found in the teacher support material.

Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.
Calculate	Obtain the answer(s) showing all relevant working. "Find" and "determine" can also be used.
Find	Obtain the answer(s) showing all relevant working. "Calculate" and "determine" can also be used.
Determine	Obtain the answer(s) showing all relevant working. "Find" and "calculate" can also be used.
Differentiate	Obtain the derivative of a function.
Integrate	Obtain the integral of a function.
Solve	Obtain the solution(s) or root(s) of an equation.
Draw	Represent by means of a labelled, **accurate** diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Sketch	Represent by means of a diagram or graph, labelled if required. A sketch should give a general idea of the required shape of the diagram or graph. A sketch of a graph should include relevant features such as intercepts, maxima, minima, points of inflexion and asymptotes.
Plot	Mark the position of points on a diagram.
Compare	Describe the similarities and differences between two or more items.
Deduce	Show a result using known information.
Justify	Give a valid reason for an answer or conclusion.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions should not generally be "analysed" using a calculator.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.

Weighting of objectives

Some objectives can be linked more easily to the different types of assessment. In particular, some will be assessed more appropriately in the internal assessment (as indicated in the following section) and only minimally in the examination papers. It is assumed that all further mathematics students are also doing mathematics HL, and will have produced a portfolio for the internal assessment component. The weightings are therefore unchanged for further mathematics SL, even though further mathematics SL students are not required to produce a portfolio.

Objective	**Percentage weighting**
Know and use mathematical concepts and principles.	15%
Read, interpret and solve a given problem using appropriate mathematical terms.	15%
Organize and present information and data in tabular, graphical and/or diagrammatic forms.	12%
Know and use appropriate notation and terminology (internal assessment).	5%
Formulate a mathematical argument and communicate it clearly.	10%
Select and use appropriate mathematical strategies and techniques.	15%
Demonstrate an understanding of both the significance and the reasonableness of results (internal assessment).	5%
Recognize patterns and structures in a variety of situations, and make generalizations (internal assessment).	3%
Recognize and demonstrate an understanding of the practical applications of mathematics (internal assessment).	3%
Use appropriate technological devices as mathematical tools (internal assessment).	15%
Demonstrate an understanding of and the appropriate use of mathematical modelling (internal assessment).	2%

