Integration and Trapezoidal Rule

Checklist

Use this space to keep track of your progress with this subtopic. Print and file this document together with those from different sub-topics for quick reference.

Task	Complete (Tick or Cross)	Traffic Light (Red, Amber, Green)
Watch the video tutorials		
Check you know your calculator skills		
Review/annotate the flashcards		
Complete the quiz		
Complete the exam style questions		
Check your solutions against the solution videos		
Review any remaining areas you need to.		

Flashcards

Find the following indefinite integrals:

1. $\int x^3 + 4x + 6\, dx = \frac{x^4}{4} + 2x^2 + 6x + c$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int k\, dx = kx + c \qquad (k \text{ is a constant})$$

2. $\int \frac{x^2}{3}\, dx = \frac{x^3}{3\times 3} + c = \frac{x^3}{9} + c$

If integrating a fractional term like this, increase the power by one first, then when you divide by the new power, multiply it by the denominator of the fraction.

3. $\int \frac{3}{x^4} dx = \int 3x^{-4} dx$

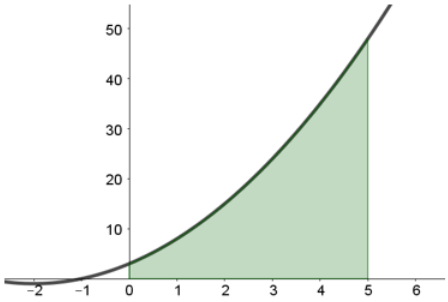
$$= \frac{3x^{-3}}{-3} + c = -x^{-3} + c = -\frac{1}{x^3} + c$$

Before you integrate a term with x^n in the denominator, you have to start by changing to power form before integrating. Also, remember to change the format back at the end.

Definite Integration: Do on the GDC

$$\int_0^5 x^2 + 4x - 3\, dx$$

This means find the area below the curve $y = x^2 + 4x - 3$ and above the x-axis, between $x = 0$ and $x = 5$.



$$\int_0^5 x^2 + 4x + 3\, dx = 106.6667\ units^2$$

Integration with a Boundary Condition

Finding the original function when you have the derivative and some information to help you find the value of *c*.
E.g.

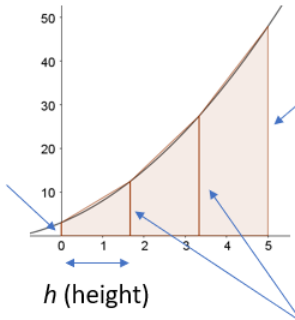
If $\frac{dy}{dx} = 3x^2 + x$, and $y = 10$ when $x = 1$, find y.

Find $f(x)$ given that $f'(x) = 3x^2 + x$ and $f(1) = 10$

1. Integrate.
2. Substitute the information given (in green above) to find the value of the constant *c*.

The Trapezoidal Rule: Estimating/Approximating the Area under a Curve

As you increase the number of trapezoids, the estimated area gets closer and closer to the exact area.



1st parallel length, y_0

Last parallel length, y_n

h (height)

Middle parallel lengths, y_1 and y_2

SL 5.8 The trapezoidal rule

$$\int_a^b y\,dx \approx \frac{1}{2}h\left((y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right),$$

where $h = \frac{b-a}{n}$

h = height of each trapezoid

n = number of trapezoids

The 1st and last parallel lengths

All of the middle parallel lengths

Exam Style Questions

Complete these questions on paper and then check your solutions against the video solutions on the website.

Question 1

A function $f(x)$ has derivative $f'(x) = 3x^2 + 12x$. The graph of f has an x-intercept at $x = -2$.

Find $f(x)$.

(6 marks)

Write answers here:

Question 2

A function, f, is given by $f(x) = -(x+2)(x-3)$

(a) Write down an integral which would find the area between the curve, f, and the x-axis, the y-axis and the line $x = 2$.

(b) Find the area of the region described in part (a).

(6 marks)

Write answers here:

Question 3

A function, f, is given by $f(x) = 3^x + 1$.

(a) Estimate the area between f and the x-axis between $1 \leq x \leq 3$ using two trapezoids.

(b) Find the exact area enlosed by f and the x-axis between $1 \leq x \leq 3$.

(c) Find the percentage error between the exact and estimated values found in parts (a) and (b).

(8 marks)

Write answers here: