Vector Product

$\boldsymbol{v} \times \boldsymbol{w}$ is a vector perpendicular to $\boldsymbol{v}$ and $\boldsymbol{w}$

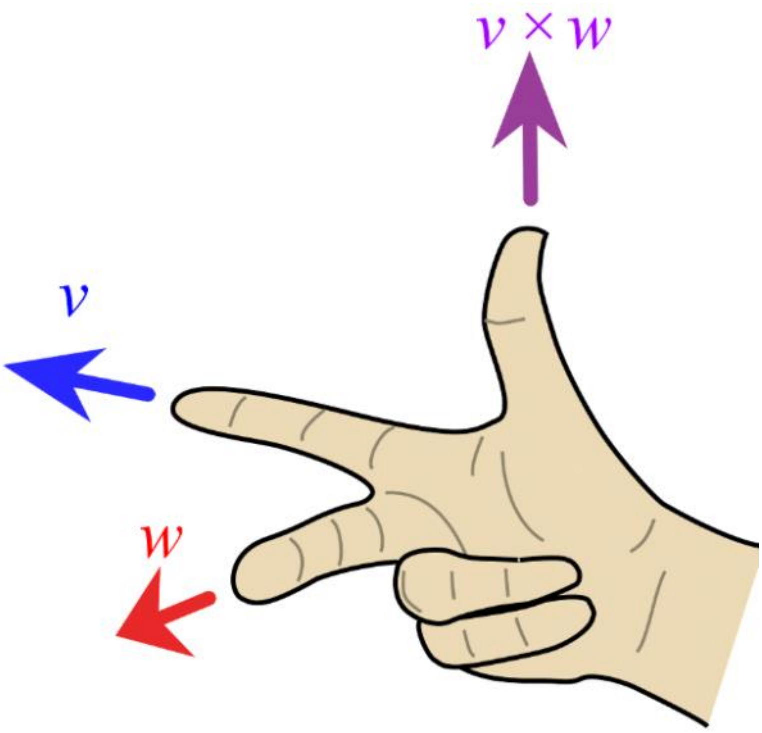
$$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ -(v_1 w_3 - v_3 w_1) \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

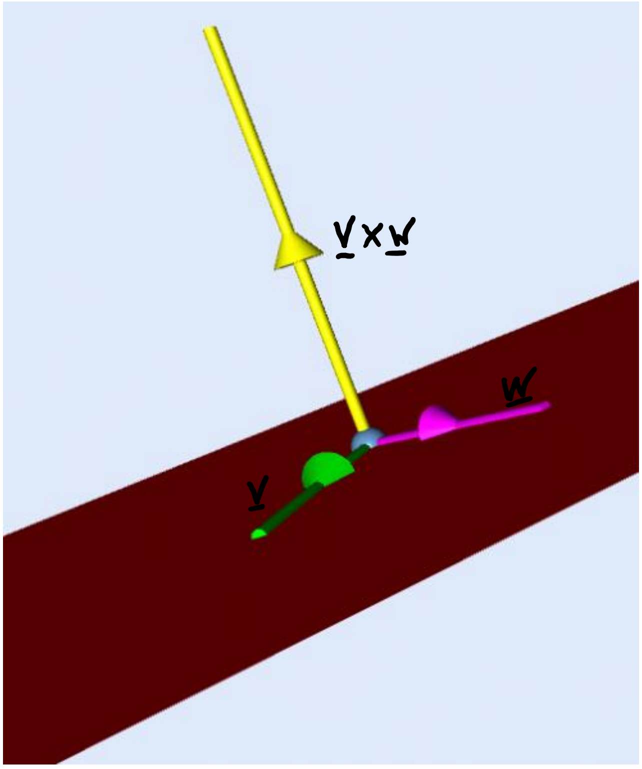
In textbooks you may see this written in another form using the discriminant of a 3by3 matrix

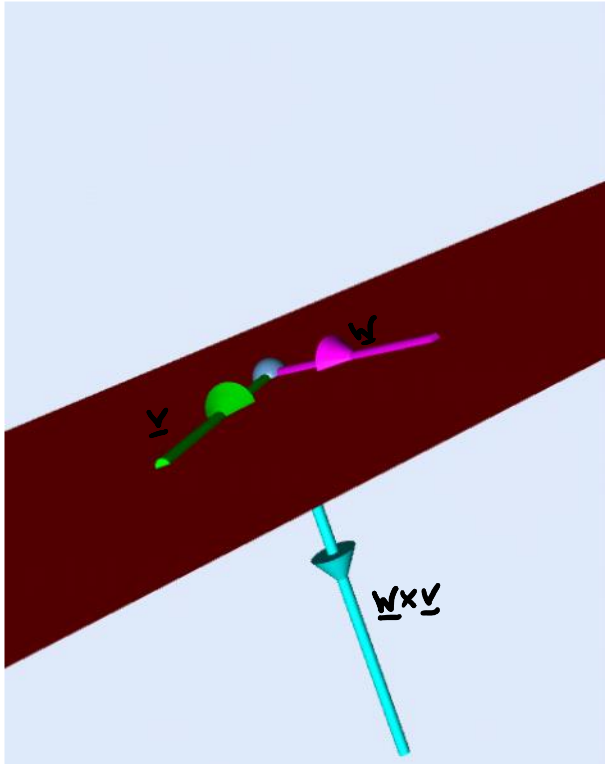
$$\boldsymbol{v} \times \boldsymbol{w} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

You can find the direction of the vector product using the ***right-hand rule***



$\boldsymbol{v} \times \boldsymbol{w}$ and $\boldsymbol{w} \times \boldsymbol{v}$ are in opposite directions





Find a vector perpendicular to $\boldsymbol{v}$ and $\boldsymbol{w}$

$$\boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$$

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ -(v_1 w_3 - v_3 w_1) \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \times 4 - 3 \times 0 \\ -(1 \times 4 - 3 \times (-1)) \\ 1 \times 0 - 2 \times (-1) \end{pmatrix} = \begin{pmatrix} 8 \\ -7 \\ 2 \end{pmatrix}$$

Check that this is perpendicular to original vectors.
Scalar products should equal **zero**

$$\begin{pmatrix} 8 \\ -7 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 8 \cdot 1 + (-7) \cdot 2 + 2 \cdot 3 = 0$$

$$\begin{pmatrix} 8 \\ -7 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = 8 \cdot (-1) + (-7) \cdot 0 + 2 \cdot 4 = 0$$

A vector perpendicular to ***v*** and ***w*** is $\begin{pmatrix} 8 \\ -7 \\ 2 \end{pmatrix}$