

a and **b** are vectors.

Show that $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = (\mathbf{ab})^2$

$$\begin{aligned} |\mathbf{a} \times \mathbf{b}| &= \mathbf{ab} \sin \theta \\ &= (\mathbf{ab} \sin \theta)^2 \\ |\mathbf{a} \times \mathbf{b}|^2 &= (\mathbf{ab})^2 \sin^2 \theta \end{aligned}$$

$$\begin{aligned} |\mathbf{a} \cdot \mathbf{b}| &= \mathbf{ab} \cos \theta \\ |\mathbf{a} \cdot \mathbf{b}|^2 &= (\mathbf{ab} \cos \theta)^2 \\ |\mathbf{a} \cdot \mathbf{b}|^2 &= (\mathbf{ab})^2 \cos^2 \theta \end{aligned}$$

$$\begin{aligned} |\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 &= (\mathbf{ab})^2 \sin^2 \theta + (\mathbf{ab})^2 \cos^2 \theta \\ &= (\mathbf{ab})^2 (\sin^2 \theta + \cos^2 \theta) && \sin^2 \theta + \cos^2 \theta \equiv 1 \\ &= (\mathbf{ab})^2 \end{aligned}$$