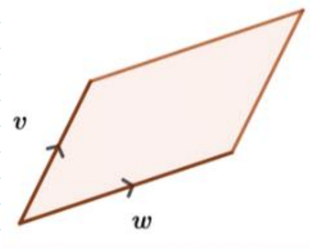
The area of a parallelogram formed by two adjacent vectors ***a*** and ***b*** is 7 square units.

$$\boldsymbol{a} = \begin{pmatrix} -3 \\ 4 \\ k \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$$

Find k



Area of parallelogram = $|\boldsymbol{v} \times \boldsymbol{w}|$

$$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{pmatrix} \times \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \mathbf{w}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_2\mathbf{w}_3 - \mathbf{v}_3\mathbf{w}_2 \\ \mathbf{v}_3\mathbf{w}_1 - \mathbf{v}_1\mathbf{w}_3 \\ \mathbf{v}_1\mathbf{w}_2 - \mathbf{v}_2\mathbf{w}_1 \end{pmatrix}$$

$$\underline{a} \times \underline{b} = \begin{pmatrix} -3 \\ 4 \\ k \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \times (-2) - (-2)k \\ -((-3)\times(-2) - 3k) \\ (-3)\times(-2) - 12 \end{pmatrix}$$

$$= \begin{pmatrix} -8 + 2k \\ 3k - 6 \\ -6 \end{pmatrix}$$

Check $\underline{a} \times \underline{b}$ is perpendicular to $\underline{a}$ and $\underline{b}$

$$\begin{pmatrix} -3 \\ 4 \\ k \end{pmatrix} \cdot \begin{pmatrix} -8 + 2k \\ 3k - 6 \\ -6 \end{pmatrix} = 24 - 6k + 12k - 24 - 6k = 0$$

$$\begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -8 + 2k \\ 3k - 6 \\ -6 \end{pmatrix} = -24 + 6k - 6k + 12 + 12 = 0$$

area of parallelogram $= |\underline{a} \times \underline{b}|$

$$= \sqrt{(-8+2k)^2 + (3k-6)^2 + (-6)^2}$$

$$= \sqrt{64 - 32k + 4k^2 + 9k^2 - 36k + 36 + 36}$$

$$= \sqrt{13k^2 - 68k + 136}$$

$$\sqrt{13k^2 - 68k + 136} = 7$$

$$13k^2 - 68k + 136 = 49$$

$$13k^2 - 68k + 87 = 0$$

$$k \approx 2.23 \quad , \quad k = 3$$