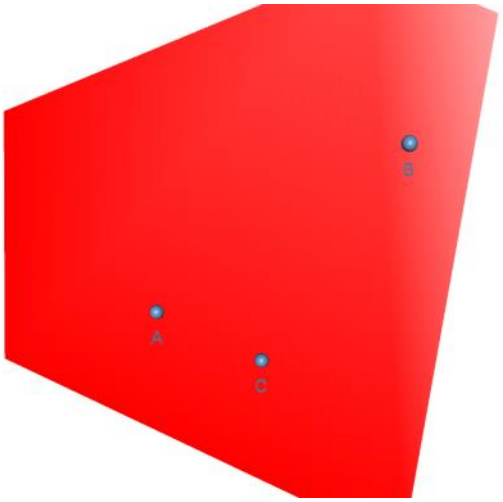
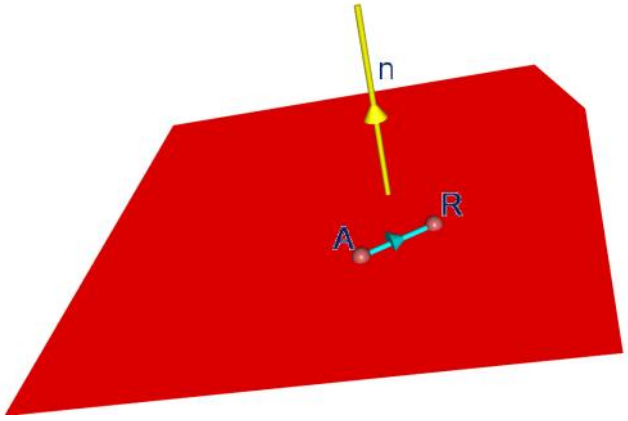
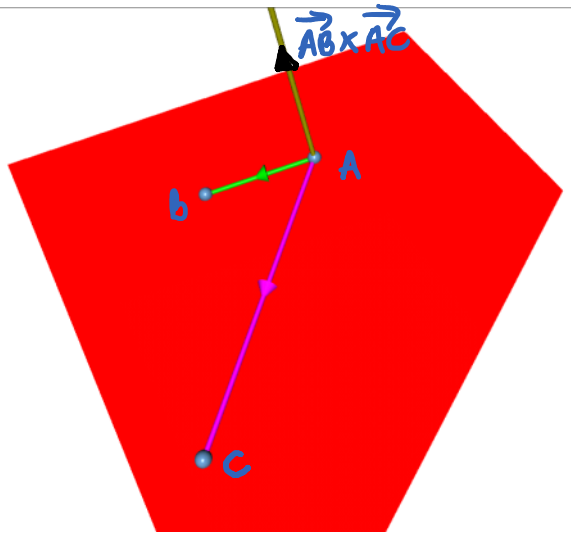
A Plane from 3 Points

Find the equation of the plane formed by the triangle A(1 , 2 , -1) , B(2 , -2 , 3) and C(0 , 2 , 1)
Does a 4th point D (1 , -1 , 2) lie in the plane?



$$\boldsymbol{r \cdot n = a \cdot n} \qquad \textit{Normal form}$$





A(1 , 2 , -1) , B(2 , -2 , 3) and C(0 , 2 , 1)

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= \begin{pmatrix}0\\2\\1\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix}$$
$$= \begin{pmatrix}-1\\0\\2\end{pmatrix}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= \begin{pmatrix}2\\-2\\3\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix}$$
$$= \begin{pmatrix}1\\-4\\4\end{pmatrix}$$

The vector product finds a vector perpendicular to 2 vectors

$$\begin{pmatrix}-1\\0\\2\end{pmatrix} \times \begin{pmatrix}1\\-4\\4\end{pmatrix} = \begin{pmatrix}0\times 4 - 2\times -4\\-(-1\times 4 - 2\times 1)\\-1\times -4 - 0\times 1\end{pmatrix}$$

$$= \begin{pmatrix}8\\6\\4\end{pmatrix}$$
$$= 2\begin{pmatrix}4\\3\\2\end{pmatrix}$$

Check this is correct by finding the scalar products

$$\begin{pmatrix}4\\3\\2\end{pmatrix} \cdot \begin{pmatrix}-1\\0\\2\end{pmatrix} = 0 \qquad \begin{pmatrix}4\\3\\2\end{pmatrix} \cdot \begin{pmatrix}1\\-4\\4\end{pmatrix} = 0$$

As the scalar products are equal to zero, the vector is perpendicular

Use the formula for the normal form

$$\boldsymbol{r}\cdot\boldsymbol{n} = \boldsymbol{a}\cdot\boldsymbol{n}$$

$$\begin{pmatrix}\boldsymbol{x}\\\boldsymbol{y}\\\boldsymbol{z}\end{pmatrix} \cdot \begin{pmatrix}4\\3\\2\end{pmatrix} = \begin{pmatrix}1\\2\\-1\end{pmatrix} \cdot \begin{pmatrix}4\\3\\2\end{pmatrix}$$

Find the scalar products to find the cartesian form

$$x\cdot 4 + y\cdot 3 + z\cdot 2 = 1\cdot 4 + 2\cdot 3 + (-1)\cdot 2$$
$$4x + 3y + 2z = 8$$

Does a D (1 , -1 , 2) lie in the plane?

$$4x + 3y + 2z = 4\cdot 1 + 3\cdot(-1) + 2\cdot 2$$
$$= 7$$
$$\neq 8$$