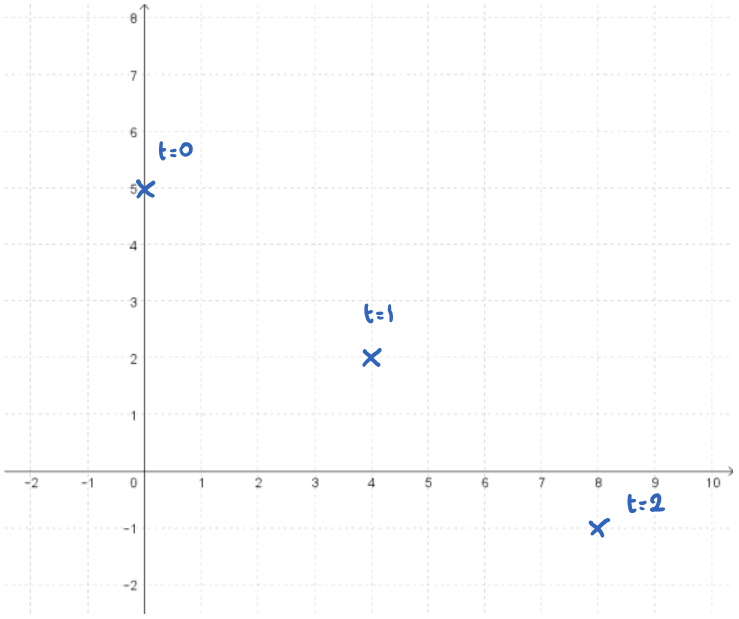
Velocity Vectors

The position, in metres, of a submarine is given by

$$\boldsymbol{r} = \begin{pmatrix}0\\5\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix}$$

where t is given in seconds

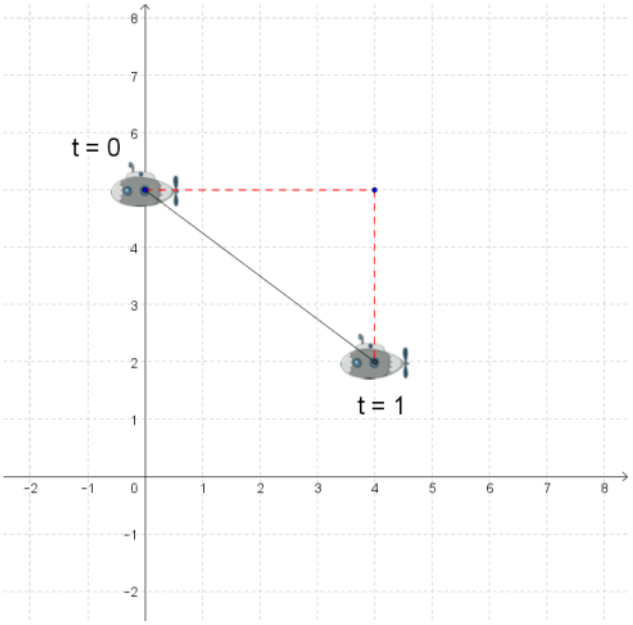


$$\boldsymbol{r} = \begin{pmatrix}0\\5\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix}$$

When t = 0, $\boldsymbol{r} = \begin{pmatrix}0\\5\end{pmatrix} + 0\begin{pmatrix}4\\-3\end{pmatrix} = \begin{pmatrix}0\\5\end{pmatrix}$

When t = 1, $\boldsymbol{r} = \begin{pmatrix}0\\5\end{pmatrix} + 1\begin{pmatrix}4\\-3\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$

When t = 2, $\boldsymbol{r} = \begin{pmatrix}0\\5\end{pmatrix} + 2\begin{pmatrix}4\\-3\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$



$$\boldsymbol{r} = \begin{pmatrix}0\\5\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix}$$

Velocity = $\begin{pmatrix}4\\-3\end{pmatrix}$ ms^{-1}

Speed = $\sqrt{4^2 + (-3)^2}$
= 5 ms^{-1}

Example

A submarine is initially positioned at (0, 5) travels with velocity $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ms^{-1}.

One second later a torpedo is fired from (3, 0) with velocity $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ms^{-1}.

Does the torpedo manage to shoot the submarine?

Submarine: $\boldsymbol{r_s} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Torpedo: $\boldsymbol{r_t} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + (t-1)\begin{pmatrix} 5 \\ 1 \end{pmatrix}$, $t > 1$

The directions are not parallel $\begin{pmatrix} 4 \\ -3 \end{pmatrix} \neq k\begin{pmatrix} 5 \\ 1 \end{pmatrix}$

This means that their paths cross.

For a collision to take place, they need to have the same position **at the same time**

$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\quad x_s = 0 + 4t$, $y_s = 5 - 3t$

$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + (t-1)\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\quad x_t = 3 + 5(t-1)$, $y_t = 0 + 1(t-1)$

Find the time when the x positions are equal

$$0 + 4t = 3 + 5(t-1)$$
$$4t = 3 + 5t - 5$$
$$4t = -2 + 5t$$
$$2 = t$$

Find the y positions at this time

$y_s = 5 - 3 \times 2 = -1$

$y_t = 0 + 1(2-1) = 2$

Since the y positions are not equal, they do not collide.