The point A (3, 1, –2) is on the line L, which is perpendicular to the plane $2x - 3y - z + 9 = 0.$

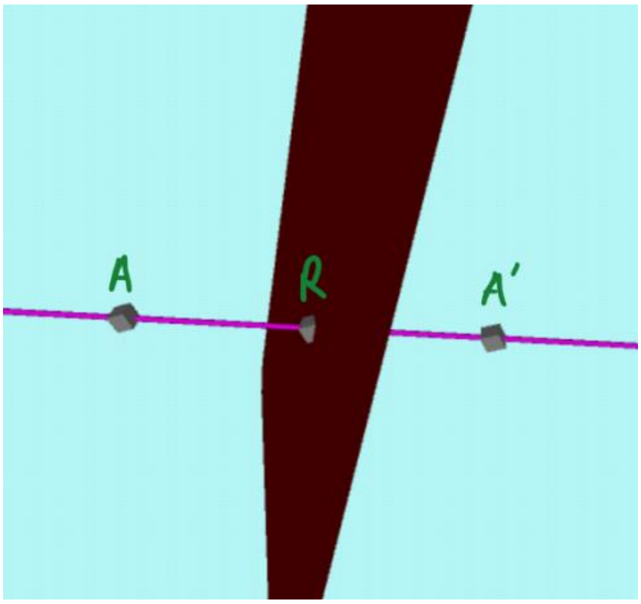
a. Find the Cartesian equation of the line L.

b. Find the point R which is the intersection of the line L and the plane.

c. The point A is reflected in the plane. Find the coordinates of the image of A.

It helps if we can visualise this situation.

R is the midpoint of A and A'



If L is perpendicular to the plane then is parallel to the normal

a)

normal $\boldsymbol{n} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

A (3, 1, –2) is on the line L

Equation of the line L $\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$$

$$x = 3 + 2\lambda$$
$$y = 1 - 3\lambda$$
$$z = -2 - \lambda$$

Cartesian Form $\dfrac{x-3}{2} = \dfrac{y-1}{-3} = \dfrac{z+2}{-1}$

b)

Find the intersection with line and plane $2x - 3y - z + 9 = 0$

$$2(3 + 2\lambda) - 3(1 - 3\lambda) - (-2 - \lambda) + 9 = 0$$

Solve for λ

$$6 + 4\lambda - 3 + 9\lambda + 2 + \lambda + 9 = 0$$

$$14\lambda = -14$$

$$\lambda = -1$$

Substitute in to equation of line

$$x = 3 + 2(-1) = 1$$
$$y = 1 - 3(-1) = 4$$
$$z = -2 - (-1) = -1$$

$\boldsymbol{R(1, 4, -1)}$

c)

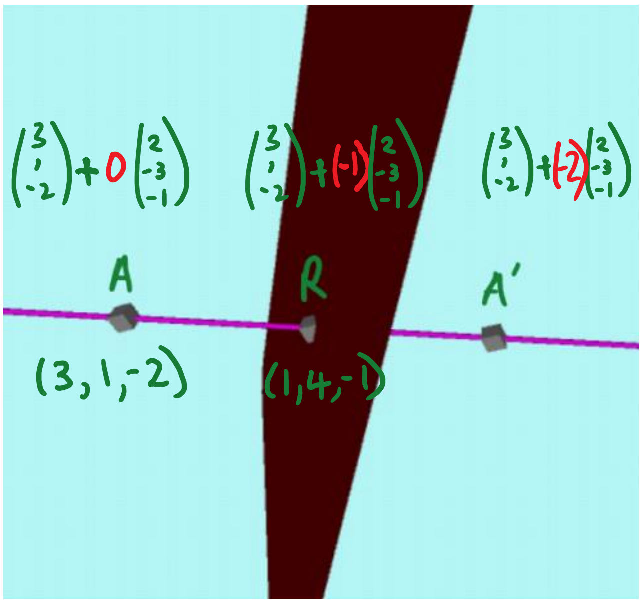
The points A , R and A' lie on the straight line. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

$\lambda = 0$ gives A

$\lambda = -1$ gives R

Therefore

$\lambda = -2$ gives A'



$$A \quad \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + 0 \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

$$R \quad \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + (-1) \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$

$$A' \quad \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + (-2) \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \\ 0 \end{pmatrix}$$

$\boldsymbol{A'(-1, 7, 0)}$