Π_1 and Π_2 are planes such that

$\Pi_1: 2x - y - 2z = 0$

and

$\Pi_2: -2x + 3y + 3z = 4$

L is the intersection of planes Π_1 and Π_2

a) Find the equation of the line L

A third plane Π_3 is defined by the equation $kx + (k-1)y - z = 5$

b) Find the value of k such that the line L **does not** intersect with Π_3

a)

$$2x - y - 2z = 0 \quad \text{A}$$

$$-2x + 3y + 3z = 4 \quad \text{B}$$

Eliminate x

A+B $\quad 2y + z = 4$

Write y in terms of z $\quad y = -0.5z + 2$

Eliminate y

3A+B $\quad 4x - 3z = 4$

Write x in terms of z $\quad x = 0.75z + 1$

So our equations become

$$x = 0.75z + 1$$

$$y = -0.5z + 2$$

$$z = z$$

Let $z = \lambda$

$$x = 0.75\lambda + 1$$

$$y = -0.5\lambda + 2$$

$$z = \lambda$$

Write in vector form

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0.75 \\ -0.5 \\ 1 \end{pmatrix}$$

Or with integer values

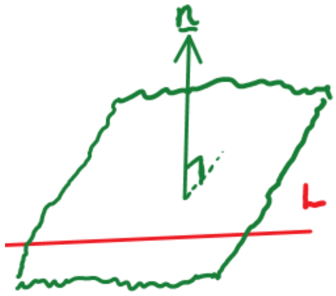
$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$$

b)

$$L: \boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$$

$\Pi_3: kx + (k-1)y - z = 5$

If the line L **does not** intersect with Π_3
Then they must be parallel



The normal to the plane Π_3 is perpendicular to L

normal $= \begin{pmatrix} k \\ k-1 \\ -1 \end{pmatrix}$

Direction of line $= \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$

Scalar product = 0 $\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} k \\ k-1 \\ -1 \end{pmatrix} = 0$

$3k - 2k + 2 - 4 = 0$

$k = 2$

Here is a graph representing the situation