A line L_1 passes through the points P(-13,-6,1) and Q(3,2,-3).

A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 9 \\ 12 \\ 2 \end{pmatrix} + s\begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$.

a) Show that $\overrightarrow{PQ} = \begin{pmatrix} 16 \\ 8 \\ -4 \end{pmatrix}$

b) Hence, write down the equation L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R.

a)

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$
$$= \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -13 \\ -6 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 16 \\ 8 \\ -4 \end{pmatrix}$$

b)

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 16 \\ 8 \\ -4 \end{pmatrix}$$

*there are other correct answers to the equation

c)

Find point of intersection of

$$L_1 : \boldsymbol{r} = \begin{pmatrix} 9 \\ 12 \\ 2 \end{pmatrix} + s\begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$$

And

$$L_2 : \boldsymbol{r} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 16 \\ 8 \\ -4 \end{pmatrix}$$

$$9 - 3s = 3 + 16t$$
$$12 + 2s = 2 + 8t$$

Simplify equations

$$3s + 16t = 6$$
$$(2s - 8t = -10) \times 2$$
$$4s - 16t = -20$$
$$3s + 16t = 6$$

Add equations to eliminate t

$$7s = -14$$
$$s = -2$$

Substitute into L_1

$$\boldsymbol{r} = \begin{pmatrix} 9 \\ 12 \\ 2 \end{pmatrix} + (-2)\begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -4 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 15 \\ 8 \\ -6 \end{pmatrix}$$

$R(15,8,-6)$