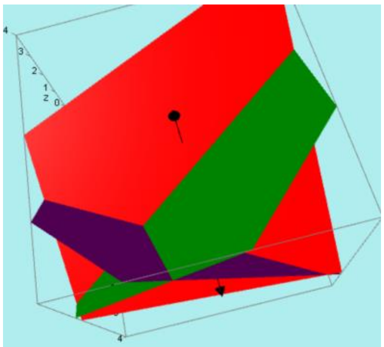
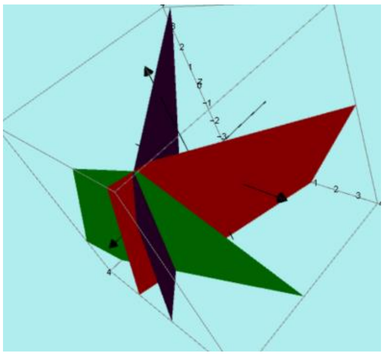
Intersection of Planes

There are three different types of solution:

Unique Solution

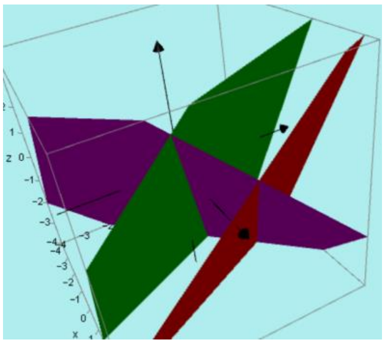


Infinite Solutions



* there are other graphical situations that give infinite solutions

Zero Solutions



** there are other graphical situations that give zero solutions

Method - Our method for solving these systems is algebraic and uses elimination

1. There are three equations with three unknowns

$$x + 3y - 2z = 7 \quad ①$$
$$2x - 2y + z = 3 \quad ②$$
$$3x + y - z = 12 \quad ③$$

2. Take two of the equations and eliminate one of the unknowns (e.g. z)

$$②+③ \qquad 5x - y = 15 \; Ⓐ$$

3. Take a different pair of equations and eliminate the same unknown (e.g. z)

$$2\times②+① \quad 5x - y = 13 \; Ⓑ$$

4. Attempt to solve equations A and B

You are left with 3 different situations depending on the number of solutions to the original system

$$5x - y = 9$$
$$3x + y = 7$$

We can solve these equations.

There is a unique solution.

$$5x - y = 15$$
$$5x - y = 13$$

We can't solve these equations, they are inconsistent.

There are zero solutions

$$5x - y = 13$$
$$5x - y = 13$$

We can solve these equations, but…

There are infinite solutions

Infinite Solutions

You are often asked to give the general solution to this. In which case you need to be able to find the equation of the straight line that the solutions usually lie on. See section on finding equation of a line in 3D.

Note, it is possible that the system represents three identical planes, but this would not come up in an exam.

Exam Questions

It is important to recognise each of the different situations above since exam questions often start with the idea that there are infinite of no solutions and you must work out the values of parameters:

Example

The system below has infinite solutions. Find ***k***.

$$\begin{aligned} 2x - y + z &= 0 \\ 3x + 2y - z &= 8 \\ x + ky + 3z &= -k^2 + 8 \end{aligned}$$

Eliminating z gives

$$\begin{aligned} 5x + \quad y &= 8 \\ 10x + (6+k)y &= 32 - k^2 \end{aligned}$$

If there are infinite solutions, then these equations are equivalent

$$\begin{aligned} 10x + \quad 2y &= 16 \\ 10x + (6+k)y &= 32 - k^2 \end{aligned}$$

Hence

$2 = 6 + k$

$k = 4$