

Find the value of k which makes the following system of equations inconsistent:

$$x + 2y + kz = -1$$

$$2x + y - z = 3$$

$$kx - 2y + z = 1$$

$$x + 2y + kz = -1 \quad A$$

$$2x + y - z = 3 \quad B$$

$$kx - 2y + z = 1 \quad C$$

Eliminate y

$$A + C \quad (1+k)x + (1+k)z = 0$$

$$B \times 2 \quad 4x + 2y - 2z = 6$$

$$C \quad kx - 2y + z = 1$$

$$B \times 2 + C \quad (4+k)x - z = 7$$

$$(1+k)x + (1+k)z = 0 \quad A + C$$

$$(4+k)x - z = 7 \quad B \times 2 + C$$

Equate coefficients of z

$$(B \times 2 + C) \times -(1+k) \quad -(1+k)(4+k)x + (1+k)z = -7(1+k) \quad (B \times 2 + C) \times -(1+k)$$

$$(1+k)x + (1+k)z = 0 \quad A + C$$

For no intersection

$$-(1+k)(4+k) = (1+k)$$

and

$$-7(1+k) \neq 0$$

$$\text{Solve } -(1+k)(4+k) = (1+k)$$

$$-4 - 5k - k^2 = 1 + k$$

$$0 = k^2 + 6k + 5$$

$$0 = (k+1)(k+5)$$

$$k = -1, k = -5,$$

$$\text{And } k = -1$$

$$-7(1+k) \neq 0 \quad -7(1+(-1)) = 0$$

$$k = -5$$

$$-7(1+(-5)) = 28 \neq 0$$

Therefore $k = -5$