

Let $f(x) = \cos x$ and $g(x) = \frac{2x^2}{1-x}$

a) Show that $g \circ f(x) = 1$ can be written as $2\cos^2 x + \cos x - 1 = 0$

b) Hence, solve $g \circ f(x) = 1$ for $-\pi \leq x \leq \pi$

a)
$$g \circ f(x) = \frac{2(\cos x)^2}{1 - (\cos x)}$$

$$g \circ f(x) = 1 \Rightarrow \frac{2\cos^2 x}{1 - \cos x} = 1$$

$$2\cos^2 x = 1 - \cos x$$

$$2\cos^2 x + \cos x - 1 = 0$$

b)
$$2\cos^2 x + \cos x - 1 = 0$$

Let $y = \cos x$

$$2y^2 + y - 1 = 0$$

$$(2y - 1)(y + 1) = 0$$

$$y = \frac{1}{2}, y = -1$$

$$\cos x = \frac{1}{2}, \cos x = -1$$

$$\text{Arccos}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Arccos}(-1) = \pi$$

Solve $-\pi \leq x \leq \pi$

$$x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$$

