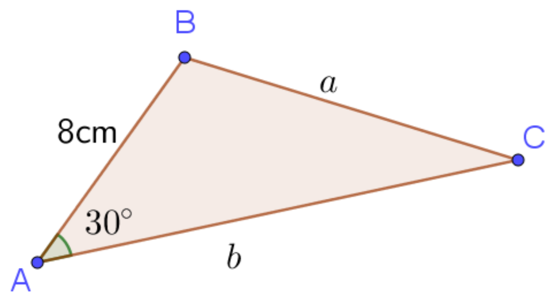
In a triangle ABC, AB = 8cm, BC = a, AC = b and $\angle BAC = 30°$

a) Show that $b^2 - 8\sqrt{3}b + 64 - a^2 = 0$

b) Hence find the possible values of a (in cm) for which the triangle has two possible solutions



Use the cosine rule:

$$a^2 = 8^2 + b^2 - 2 \times 8 \times b \times cos30°$$

$$a^2 = 64 + b^2 - 16b \times \frac{\sqrt{3}}{2}$$

$$a^2 = 64 + b^2 - 8\sqrt{3}b$$

$$b^2 - 8\sqrt{3}b + 64 - a^2 = 0$$

Use the quadratic formula or complete the square:

$$b = \frac{8\sqrt{3} \pm \sqrt{\left(8\sqrt{3}\right)^2 - 4 \times 1(64 - a^2)}}{2}$$

$$b = \frac{8\sqrt{3} \pm \sqrt{192 - 4(64 - a^2)}}{2}$$

Note that dividing by 2 is like dividing by $\sqrt{4}$

$$b = 4\sqrt{3} \pm \sqrt{48 - 64 + a^2}$$

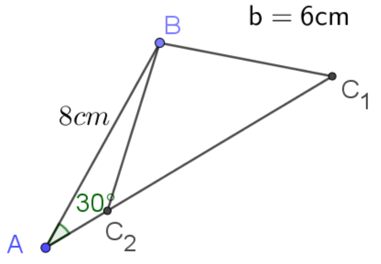
$$b = 4\sqrt{3} \pm \sqrt{a^2 - 16}$$

This equation has

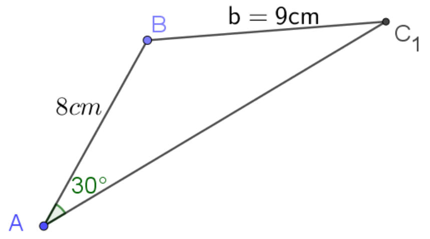
- Zero roots when $a^2 - 16 < 0 \quad \Rightarrow a < 4$
- One root when $a^2 - 16 = 0 \quad \Rightarrow a = 4$
- Two roots when $a^2 - 16 > 0 \quad \Rightarrow a > 4$

For example,

when b = 6cm, there are two solutions



when b = 9cm, there is one solution



So the triangle appears to have two possible solutions when $a > 4$

…except that there is an upper limit to this

If $a \geq 8$, the triangle has one solution

The triangle has two possible solutions for

$\mathbf{4 < a < 8}$