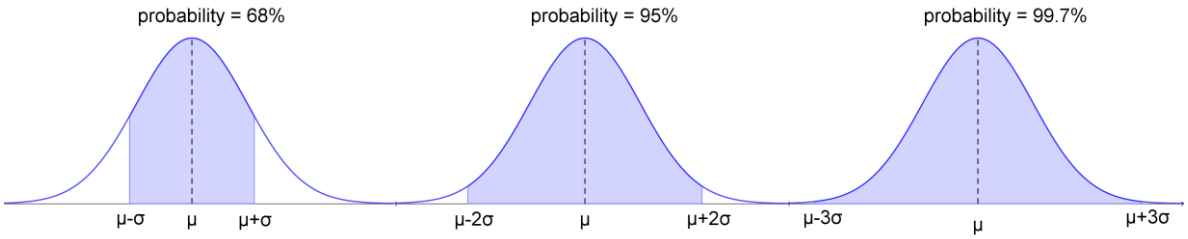
The Normal Distribution

The Normal Distribution is a continuous probability distribution. For a random variable X that is normally distributed with $\boldsymbol{\mu}$ = mean and $\boldsymbol{\sigma^2}$ =variance, we write $\boldsymbol{X \sim N(\mu, \sigma^2)}$

Probabilities are found by calculating the areas under a bell-shaped graph.

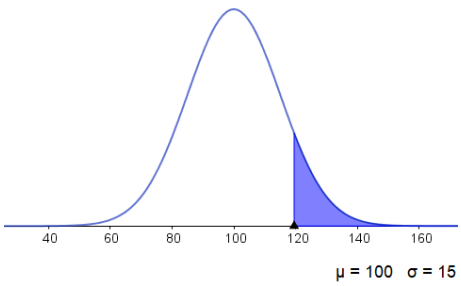
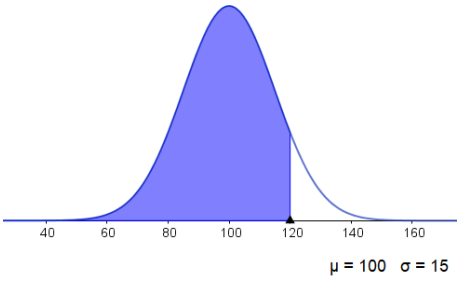
- 68% of the data are within 1 standard deviation of the mean
- 95% of the data are within 2 standard deviations of the mean
- 99.7% of the data are within 3 standard deviation of the mean



There are 3 types of question you could be asked. In each case, you should draw sketches of the graph to help you visualise the problem:

1. Finding Probabilities

- On your calculator, use **normalcdf**

$\boldsymbol{X \sim N(100, 15^2)}$	$\boldsymbol{X \sim N(100, 15^2)}$
	
$P(X > 120) \approx 0.0912$ Lower = 120 Upper =9×10^{99} μ=100 σ=15*	$P(X < 120) \approx 0.909$ Lower = -9×10^{99} Upper =120 μ=100 σ=15

* Be careful to enter standard deviation and not the variance!

2. Inverse Problems

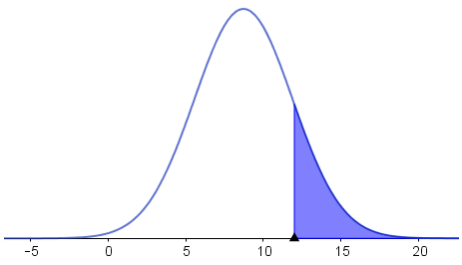
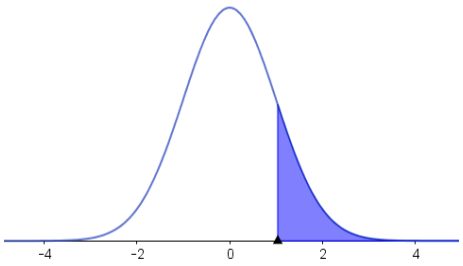
On your calculator, use **InvNorm**

3a. Finding Mean OR Standard Deviation

- Use the Standard Normal Distribution $\boldsymbol{Z \sim N(0, 1^2)}$ to standardise your random variable using $\boldsymbol{Z = \frac{X-\mu}{\sigma}}$

Example

Find $\boldsymbol{\mu}$ if $\boldsymbol{X \sim N(\mu, 10)}$ and $P(X > 12) = 0.15$

$\boldsymbol{X \sim N(\mu, 10)}$	$\boldsymbol{Z \sim N(0, 1^2)}$
 12	 Z_1 Use InvNorm to find $Z_1 \approx 1.0364$

$$1.0364 \approx \frac{12 - \mu}{\sqrt{10}}$$

$$\mu \approx 12 - 1.0364 \times \sqrt{10}$$

3b. Finding Mean AND Standard Deviation

- In this type of question, you will be given two pieces of information about probabilities. Carry out the question as above to find two equations with two unknowns.
- Solve the equations using the simultaneous equation solver on your calculator.