The probability density function of X is given by

$$f(x) = \begin{cases} ax^n & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $a = n + 1$

b) Given that $E(X) = \frac{3}{4}$

---

a) For continuous random variables, the total area under the function = 1

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

In our case,

$$\int_0^1 ax^n dx = 1$$

$$\left[\frac{ax^{n+1}}{\mathrm{n}+1}\right]_0^1 = 1$$

$$\frac{a}{\mathrm{n}+1} = 1$$

Therefore, $\mathrm{a} = \mathrm{n} + 1$

b) For continuous random variables,

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} xf(x)dx$$

In our case,

$$E(X) = \int_0^1 x \cdot ax^n dx$$

$$E(X) = \int_0^1 ax^{n+1} dx$$

We know from part a) that $\mathrm{a} = \mathrm{n} + 1$

$$E(X) = \int_0^1 (\mathrm{n}+1)x^{n+1} dx$$

$$E(X) = \left[\frac{(\mathrm{n}+1)x^{n+2}}{\mathrm{n}+2}\right]_0^1$$

$E(X) = \frac{\mathrm{n}+1}{\mathrm{n}+2}$

We are told that $E(X) \ = \frac{3}{4}$

$\frac{\mathrm{n}+1}{\mathrm{n}+2} = \frac{3}{4}$

Therefore, $n = 2$

And $\mathrm{a} \ = \ 3$