

Homogeneous Differential Equations

A first order differential equation is homogeneous if it can be written in the following form

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

We may need to re-arrange our differential equation to get it into the recognisable form

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

There are several steps involved in this method...

1. Substitute $v = \frac{y}{x}$
2. And $\frac{dy}{dx} = v + x \frac{dv}{dx}$
3. Rearrange the equation. It should become a variables separable differential equation in the form

$$\int f(v)dv = \int f(x) dx$$

4. Carry out the integration and simplify your result
5. Substitute back $\frac{y}{x} = v$
6. Simplify