Consider a function $f(x)$ such that $\int_0^4 f(x)dx = 6$

Find

a) $\int_0^4 3f(x)dx$

b) $\int_0^4 [f(x) + 3]dx$

c) $\int_{-3}^{1} \frac{1}{3} f(x+3)dx$

d) $\int_0^4 [f(x) + x]dx$

This question is all about properties of the definite integral

$$\int_a^b \boldsymbol{k} f(x)dx = \boldsymbol{k} \int_a^b f(x)dx$$

a) $$\int_0^4 3f(x)dx = 3\int_0^4 f(x)dx = 3 \times 6 = \mathbf{18}$$

$$\int_a^b \big(f(x) \pm g(x)\big)dx$$

$$= \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

b) $$\int_0^4 [f(x) + 3]dx$$

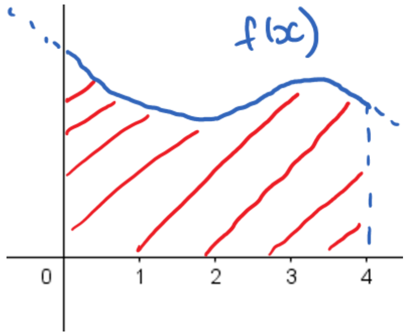
$$= \int_0^4 f(x)dx + \int_0^4 3dx$$

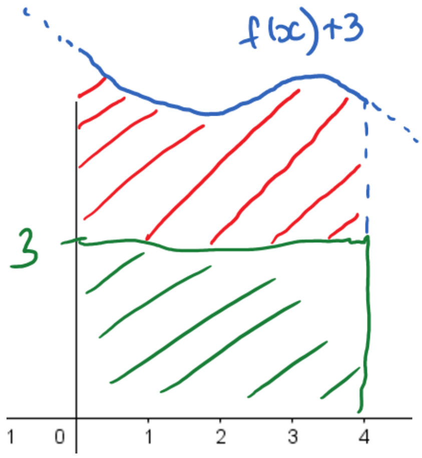
$$= 6 + [3x]_0^4$$

$$= 6 + 12$$

$$= \mathbf{18}$$

You can also think about this question as translating the graph 3 units up. It creates a $4\ by\ 3$ rectangle below the original graph.

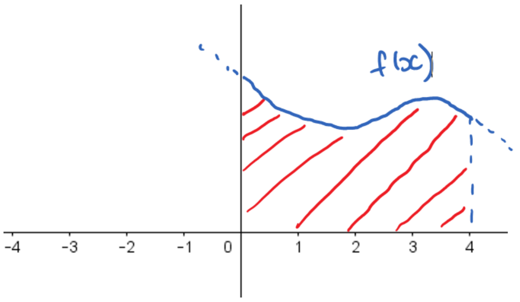


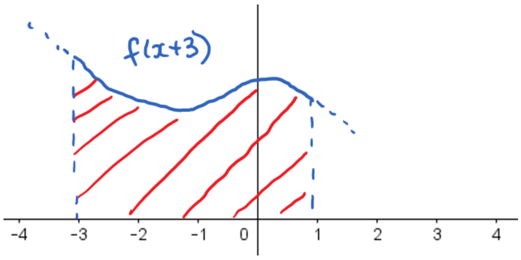


$= 6 + 12$

$= \mathbf{18}$

$f(x+3)$ translates the graph of $f(x)$ 3 units to the left





$$\int_{-3}^{1} f(x+3)dx = \int_{-3+3}^{1+3} f(x)dx$$

c) $$\int_{-3}^{1} \frac{1}{3} f(x+3)dx$$

$$= \frac{1}{3}\int_{-3}^{1} f(x+3)dx$$

$$= \frac{1}{3}\int_{0}^{4} f(x)dx$$

$$= \frac{1}{3} \times 6$$

$$= \mathbf{2}$$

d) $$\int_{0}^{4} [f(x)+x]dx$$

$$= \int_{0}^{4} f(x)dx + \int_{0}^{4} xdx$$

$$= 6 + \left[\frac{x^2}{2}\right]_0^4$$

$$= 6 + 8$$

$$= \mathbf{14}$$