Integration by Substitution

This is a technique for integrating more complicated functions. Some people think of it as the reverse chain rule and it is certainly useful to be confident with that technique first!

The key to this integration by substitution (or U-substitution) is recognizing that you need to use it and what the substitution is. In the exam, it is rare that the substitution is suggested for you and so it is important that you get lots of practice in using it.

$$\int kg'(x)f[g(x)]dx$$

$$u = g(x)$$

Usually, we are looking for a function, $g(x)$,within part of the integrand, that when we differentiate it, we also see in the integrand, $g'(x)$

There are some patterns that will help you and to get you started recognizing the substitution, here are some questions with the suggested substitution.

Question	Substitution
$\int 6x^2[x^3+2]^4dx = \int 2\cdot 3x^2[x^3+2]^4dx$	$u = x^3+2$ $\frac{du}{dx} = 3x^2$
$\int sinxcos^2xdx = \int -1(-sinx)[cosx]^2dx$	$u = cosx$ $\frac{du}{dx} = -sinx$
$\int \frac{cosx}{sinx}dx = \int (cosx)\frac{1}{sinx}dx$	$u = sinx$ $\frac{du}{dx} = cosx$
$\int \frac{cosx}{sin^3x}dx = \int (cosx)\frac{1}{sin^3x}dx$	$u = sinx$ $\frac{du}{dx} = cosx$
$\int \frac{(\sqrt{x}-1)^2}{\sqrt{x}}dx = \int 2\left(\frac{1}{2\sqrt{x}}\right)(\sqrt{x}-1)^2dx$	$u = \sqrt{x}-1 = x^{0.5}-1$ $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

Refer to the videos on the integration by substitution page for more complicated examples.