Exam-style Question

$$\int 2x \cdot arctanx\, dx =$$

Use integration by parts:

$$\int u \cdot \frac{dv}{dx}\, dx \;=\; uv \;-\; \int v \cdot \frac{du}{dx}\, dx$$

$u = \arctan x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$\frac{dv}{dx} = 2x$

$v = x^2$

$$\int 2x \cdot arctanx\, dx = (\arctan x)(x^2) - \int (x^2)\left(\frac{1}{1+x^2}\right) dx$$

$$= x^2 \arctan x - \int \frac{x^2}{1+x^2}\, dx$$

$$\begin{array}{r|l} & \;1 \\ 1+x^2 & x^2+0 \\ & x^2+1 \\ \hline & \quad\; -1 \end{array}$$

$$\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$= x^2 \arctan x - \int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= x^2 \arctan x - x + \arctan x \quad + C$$