

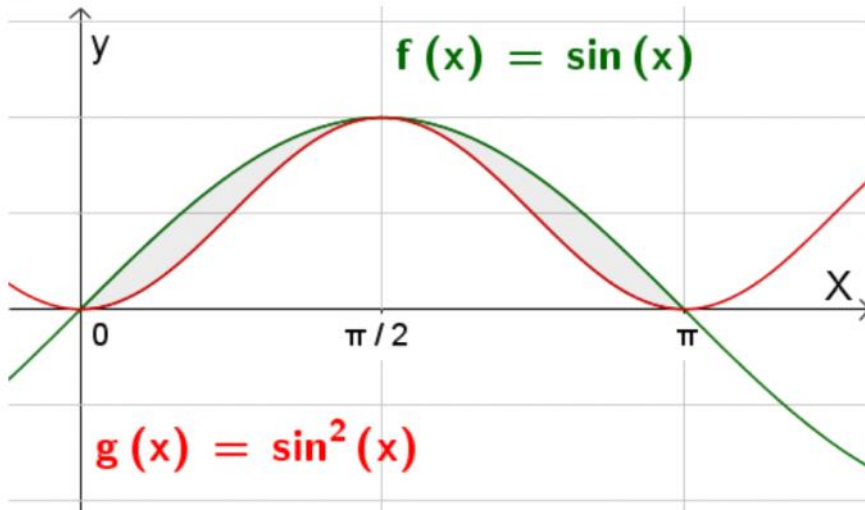
Area between Graphs



Show that the area bounded by the graphs of $y = f(x)$ and $y = g(x)$ in the interval $0 \leq x \leq \pi$ is given by $2 - \frac{\pi}{2}$

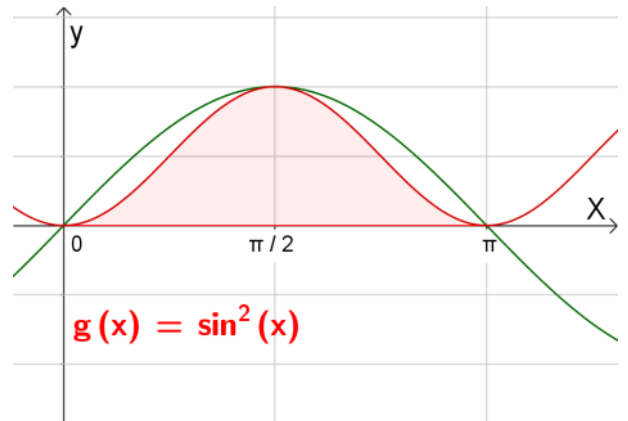
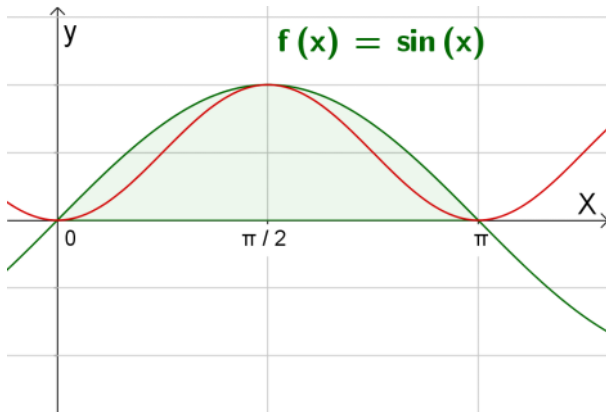
$f(x) = \sin x$

$g(x) = \sin^2 x$



Area under a curve

$$A = \int_a^b y \, dx \text{ or } A = \int_a^b x \, dy$$



$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

Double angle identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \int_0^{\pi} \sin x \, dx - \int_0^{\pi} \sin^2 x \, dx$$

$$= [-\cos x]_0^{\pi} - \int_0^{\pi} \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$$

$$= [-\cos \pi - (-\cos 0)] - \left[\frac{1}{2} x - \frac{1}{4} \sin 2x \right]_0^{\pi}$$

$$= [1 + 1] - \left[\left(\frac{\pi}{2} - \frac{1}{4} \sin 2\pi \right) - \left(0 - \frac{1}{4} \sin 0 \right) \right]$$

$$= 2 - \left[\left(\frac{\pi}{2} - 0 \right) - (0 - 0) \right]$$

$$= 2 - \frac{\pi}{2}$$

$$\cos 2x \equiv 1 - 2 \sin^2 x$$

$$2 \sin^2 x \equiv 1 - \cos 2x$$

$$\sin^2 x \equiv \frac{1}{2} - \frac{1}{2} \cos 2x$$