Exam-style Question – Area between Graphs

a) f(x)= $\ln\left(\frac{x-1}{x}\right)$, x>1

Using log laws, we can make this function much easier to differentiate:
f(x) = ln(x-1) – ln(x)

Differentiating with respect to x
f'(x) = $\frac{1}{x-1} - \frac{1}{x}$

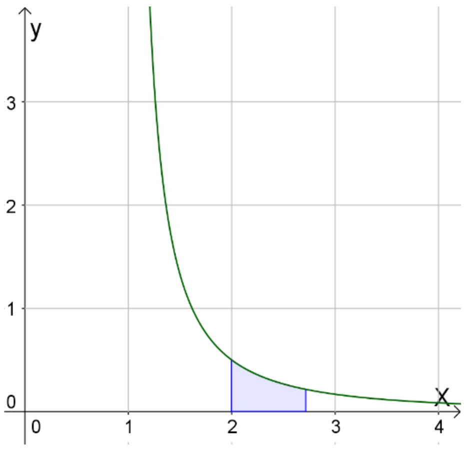
Simplifying:
f'(x) = $\frac{x}{x(x-1)} - \frac{x-1}{x(x-1)}$

f'(x) = $\frac{x-x+1}{x(x-1)}$

f'(x) = $\frac{1}{x(x-1)}$

b) In this part, we are calculating the area bounded by g(x) = $\frac{1}{x(x-1)}$, the x axis, x=2 and x = e

Area = $\int_2^e \frac{1}{x(x-1)}dx$



Notice that integral is the answer we found in part a)

Area $= \int_2^e \frac{1}{x(x-1)}dx$
$= \left[ln\left(\frac{x-1}{x}\right)\right]_2^e$
$= \left[ln\left(\frac{e-1}{e}\right)\right] - \left[ln\left(\frac{2-1}{2}\right)\right]$
$= \left[ln\left(\frac{e-1}{e}\right)\right] - \left[ln\left(\frac{1}{2}\right)\right]$
$= \left[ln\left(\frac{e-1}{e}\right)\right] + ln2$
$= ln2\left(\frac{e-1}{e}\right)$
$= ln\left(\frac{2e-2}{e}\right)$

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