Consider the equation $64x^3 - 144x^2 + 92x - 15 = 0$

a. Write down the numerical value of the sum and the product of the roots of this equation.
b. The roots of this equation are three consecutive terms of an arithmetic sequence. Solve the equation.

a.

$$64x^3 - 144x^2 + 92x - 15 = 0$$

Sum of the roots $= -\frac{-144}{64}$

$$= \frac{9}{4}$$

Product of the roots $= -\frac{-15}{64}$

$$= \frac{15}{64}$$

b.

Let the three roots be α, β, γ

$+d \quad +d$

The three roots form terms of an arithmetic sequence α, β, γ

We could write the terms as $\beta - d, \beta, \beta + d$

Sum of the roots $\beta - d + \beta + \beta + d = 3\beta$

$$3\beta = \frac{9}{4}$$

$$\beta = \frac{3}{4}$$

Roots are $\frac{3}{4} - d \quad , \quad \frac{3}{4} \quad , \quad \frac{3}{4} + d$

Product of the roots $\left(\frac{3}{4} - d\right)\frac{3}{4}\left(\frac{3}{4} + d\right) = \frac{15}{64}$

$$\frac{3}{4}\left(\frac{3}{4} - d\right)\left(\frac{3}{4} + d\right) = \frac{15}{64}$$

$$\left(\frac{3}{4} - d\right)\left(\frac{3}{4} + d\right) = \frac{5}{16}$$

$$\frac{9}{16} - d^2 = \frac{5}{16}$$

$$\frac{9}{16} - \frac{5}{16} = d^2$$

$$\frac{1}{4} = d^2$$

$$d = \pm\frac{1}{2}$$

Roots are $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}$