The quadratic equation $x^2 - 4x + 5 = 0$ has roots α and β.

a. Without solving the equation, find the value of
   i. $\alpha + \beta$;
   ii. $\alpha\beta$.

b. Another quadratic equation $5x^2 + bx + c = 0, b, c \in \mathbb{Z}$, has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
   Find the value of $b$ and the value of $c$.

a.

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

Sum of roots = $-\frac{a_{n-1}}{a_n}$

Product of roots = $(-1)^n \frac{a_0}{a_n}$

$$1x^2 - 4x + 5 = 0$$

$\alpha + \beta = -\frac{-4}{1} = 4$

$\alpha\beta = (-1)^2 \frac{5}{1} = 5$

b.

Another quadratic equation has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sum of roots = $\frac{1}{\alpha} + \frac{1}{\beta}$

Product of roots= $\frac{1}{\alpha} \times \frac{1}{\beta}$

$$\begin{aligned}\text{Sum of roots} &= \frac{1}{\alpha} + \frac{1}{\beta} \\ &= \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta} \\ &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \frac{1}{\alpha} \times \frac{1}{\beta} \\ &= \frac{1}{\alpha\beta} \\ &= \frac{1}{5}\end{aligned}$$

$$5x^2 + bx + c = 0$$

Sum of roots= $\frac{-b}{5} = \frac{4}{5}$

$b = -4$

Product of roots= $\frac{c}{5} = \frac{1}{5}$

$c = 1$