The quadratic equation $3x^2 - 8x + 2 = 0$ has roots α and β.

a. Without solving the equation, find the value of $\alpha + \beta$ and $\alpha\beta$.

b. Another quadratic equation $3x^2 + bx + c = 0$, $b, c \in \mathbb{Z}$ has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. Find the value of b and the value c

a.

$$3x^2 - 8x + 2 = 0$$

Sum of roots $\alpha + \beta = \frac{8}{3}$

Product of roots $\alpha\beta = \frac{2}{3}$

b.

$$3x^2 + bx + c = 0$$

Sum of roots $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2}{\alpha\beta} + \frac{\beta^2}{\alpha\beta}$

$$\frac{\alpha^2}{\alpha\beta} + \frac{\beta^2}{\alpha\beta} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{8}{3}\right)^2 - 2\left(\frac{2}{3}\right)}{\frac{2}{3}}$$

$$= \frac{\frac{64}{9} - \frac{4}{3}}{\frac{2}{3}}$$

$$= \frac{\frac{64}{9} - \frac{12}{9}}{\frac{2}{3}}$$

$$= \left(\frac{52}{9}\right) \times \frac{3}{2}$$

$$= \frac{26}{3}$$

Product of roots $= \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$

$$3x^2 + bx + c = 0$$

Sum of roots $\frac{-b}{3} = \frac{26}{3} \Rightarrow b = -26$

Product of roots $\frac{c}{3} = 1 \Rightarrow c = 3$