One root of the equation $4z^4 - 4z^3 - 25z^2 + 55z - 42 = 0$ is $1 + \frac{\sqrt{3}}{2}i$.
Find the other roots of the equation.

If $z = 1 + \frac{\sqrt{3}}{2}i$ is a root ...then, the complex conjugate $z^* = 1 - \frac{\sqrt{3}}{2}i$ is also a root

Factors of the equation are

$$z - \left(1 + \frac{\sqrt{3}}{2}i\right)$$

$$z - \left(1 - \frac{\sqrt{3}}{2}i\right)$$

The polynomial equation is

$$a(x-\alpha)(x-\beta)\left(z - \left(1 + \frac{\sqrt{3}}{2}i\right)\right)\left(z - \left(1 - \frac{\sqrt{3}}{2}i\right)\right) = 0$$

$$\left(z - \left(1 + \frac{\sqrt{3}}{2}i\right)\right)\left(z - \left(1 - \frac{\sqrt{3}}{2}i\right)\right)$$

$$= z^2 - \left(1 - \frac{\sqrt{3}}{2}i\right)z - \left(1 + \frac{\sqrt{3}}{2}i\right)z + \left(1 + \frac{\sqrt{3}}{2}i\right)\left(1 - \frac{\sqrt{3}}{2}i\right)$$

$$= z^2 - 2z + 1 - \frac{3}{4}i^2$$

$$= z^2 - 2z + \frac{7}{4}$$

$$4z^4 - 4z^3 - 25z^2 + 55z - 42 = a(z-\alpha)(z-\beta)\left(z^2 - 2z + \frac{7}{4}\right)$$

$$= (z-\alpha)(z-\beta)(4z^2 - 8z + 7)$$

$$= (z^2 + bz - 6)(4z^2 - 8z + 7)$$

$-4z^3 = -8z^3 + 4bz^3$ $b = 1$

$$4z^4 - 4z^3 - 25z^2 + 55z - 42 = (z^2 + z - 6)(4z^2 - 8z + 7)$$

$$4z^4 - 4z^3 - 25z^2 + 55z - 42 = 0$$

$$4z^2 - 8z + 7 = 0$$

$$z = 1 + \frac{\sqrt{3}}{2}i, z = 1 - \frac{\sqrt{3}}{2}i$$

$$z^2 + z - 6 = 0$$

$$(z+3)(z-2) = 0$$

$$z = -3, z = 2$$

Other roots $z = 1 - \frac{\sqrt{3}}{2}i, z = -3, z = 2$