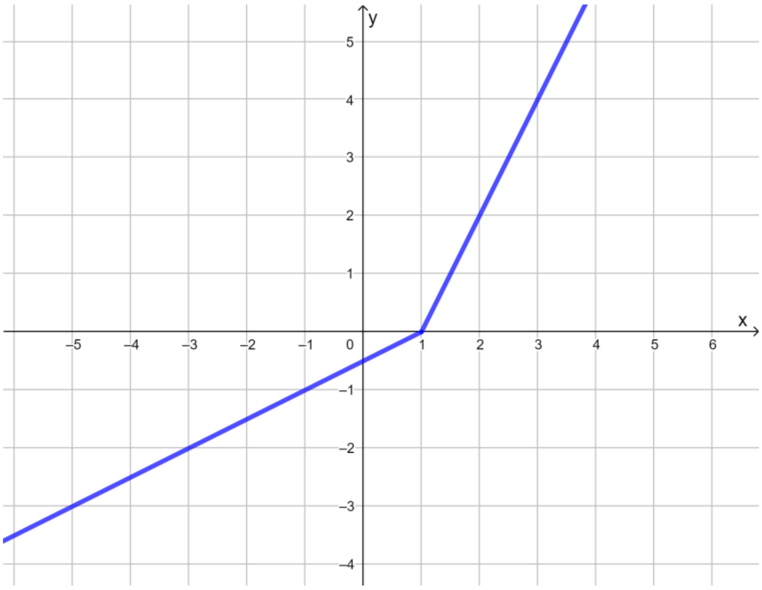
Let $f(x) = \frac{ax-1}{x-a}, x \neq a$

a) Show that $f^{-1}(x) = f(x)$

b) Hence, find $ff(b)$

c) The graph of $y = g(x)$ is shown below. Find the values of ***a*** if $fg(3) = a + 5$



d) Draw the graph of $y = g^{-1}(x)$

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a) $$y = \frac{ax-1}{x-a}$$

Interchange x and y

$$x = \frac{ay-1}{y-a}$$

Make ***x*** the subject

$$\mathrm{x(y-a) = ay - 1}$$

$$xy - ax = ay - 1$$

$$xy - ay = ax - 1$$

$$y(x-a) = ax - 1$$

$$y = \frac{ax-1}{x-a}$$

$$f^{-1}(x) = \frac{ax - 1}{x - a}$$

Hence, $f^{-1}(x) = f(x)$

Note ***f*** is a self-inverse function:

b) $f^{-1}f(x) = x$

Since, $f^{-1}(x) = f(x)$

...then $ff(x) = x$

$ff(b) = b$

c) From graph, $g(3) = 4$

Therefore, $fg(3) = f(4)$

If $fg(3) = a + 5$, then $f(4) = a + 5$

$$f(4) = \frac{4a - 1}{3 - a}$$

Solve

$$\frac{4a - 1}{3 - a} = a + 5$$

$4a - 1 = (a + 5)(3 - a)$

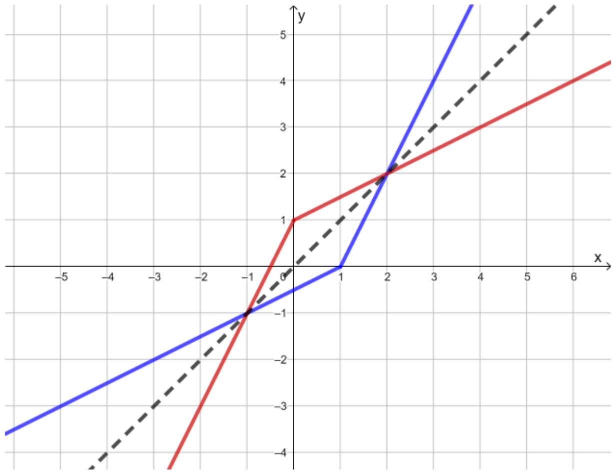
$4a - 1 = 3a - a^2 + 15 - 5a$

$a^2 + 6a - 16 = 0$

$(a + 8)(a - 2) = 0$

$a = -8, a = 2$

d) The inverse function is the reflection in the line y = x


y
x
5
4
3
2
1
0
−1
−2
−3
−4
−5
−4
−3
−2
−1
1
2
3
4
5
6