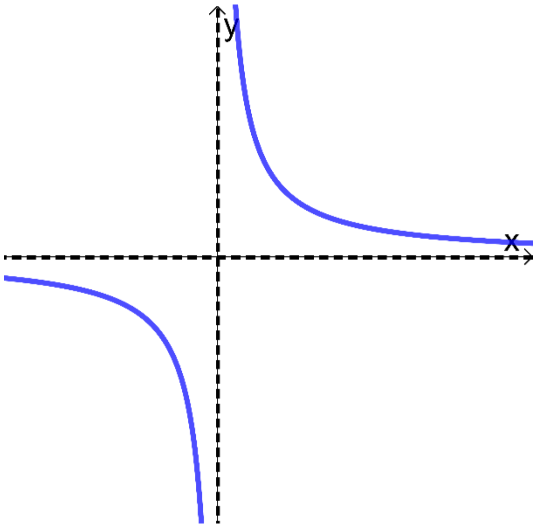
Rational Functions

The Reciprocal Function

$f(x) = \frac{1}{x}, x \neq 0$

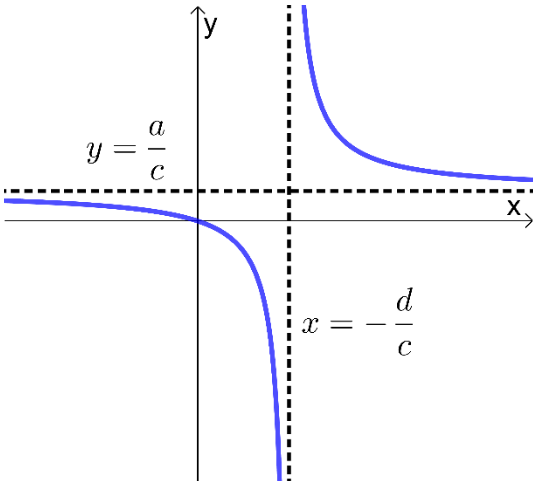


The domain of the function is $x \in \mathbb{R}, x \neq 0$
The range of the function is $f(x) \in \mathbb{R}, f(x) \neq 0$

The graph has

- a vertical asymptote at x = 0
- a horizontal asymptote at y = 0

The Rational Function Case 1: $f(x) = \frac{ax+b}{cx+d}, x \neq -\frac{d}{c}$



The domain of the function is $x \in \mathbb{R}, x \neq -\frac{d}{c}$
The range of the function is $f(x) \in \mathbb{R}, f(x) \neq \frac{a}{c}$

The graph has

- a vertical asymptote at $x = -\frac{d}{c}$
- a horizontal asymptote at $y = \frac{a}{c}$

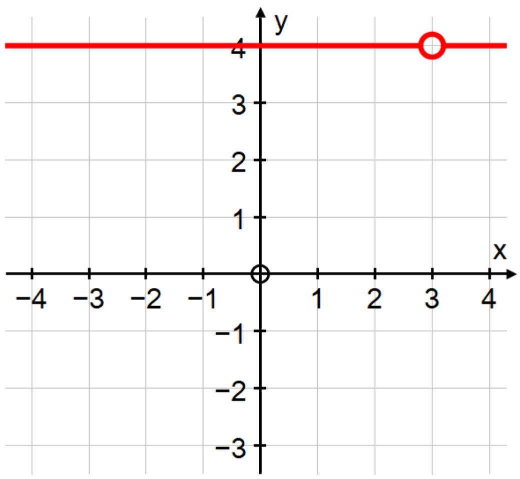
The graph does not have any stationary points

Make sure you include the x and y intercepts in sketches.

Special Function – the hole

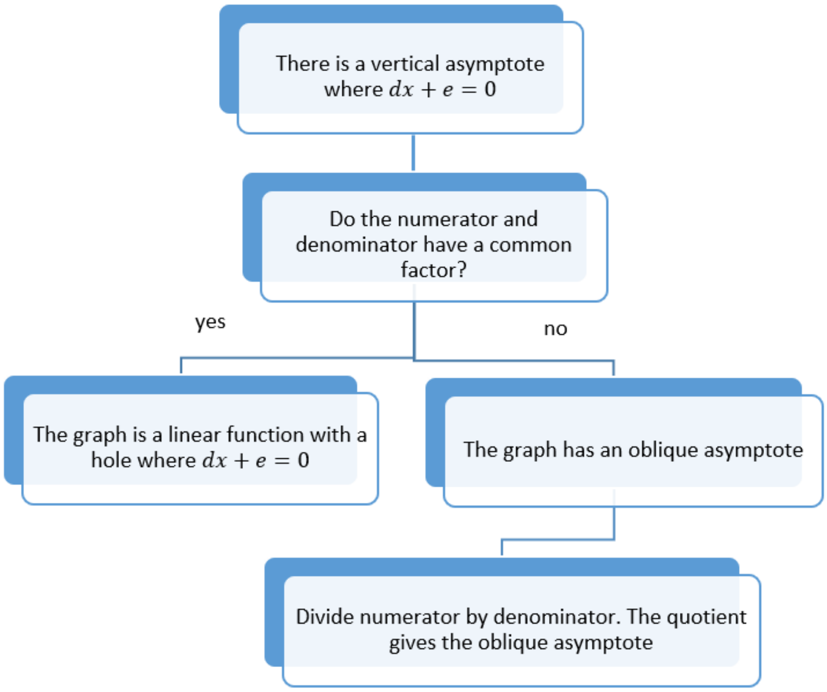
If the numerator and denominator have a common linear factor, then the graph of the function is a horizontal line with a hole

e.g. $f(x) = \frac{4(x-3)}{x-3}, x \neq 3$



The Rational Function Case 2: $f(x) = \frac{ax^2+bx+c}{dx+e}, x \neq -\frac{e}{d}$

The process to go through to find any asymptotes and sketch the graph is as follows

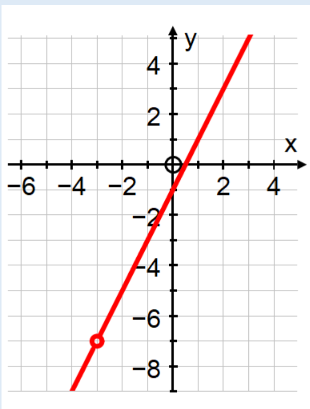


$$f(x) = \frac{2x^2 + 5x - 3}{x + 3}, x \neq -3$$

$$f(x) = \frac{(2x-1)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$f(x) = 2x - 1, \quad x \neq -3$$

Vertical asymptote at $x = -3$

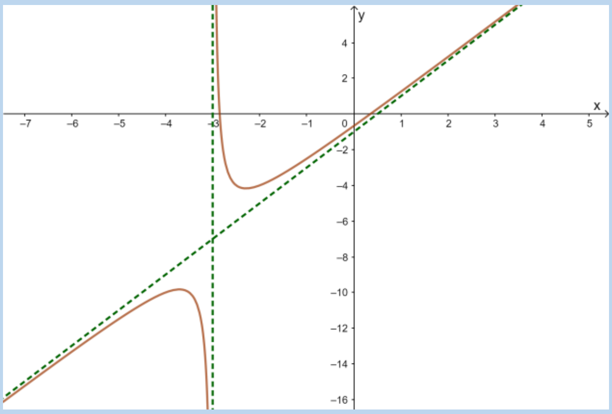


$$f(x) = \frac{2x^2 + 5x - 2}{x + 3}, x \neq -3$$

$$f(x) = 2x - 1 + \frac{1}{x+3}, x \neq -3$$

Vertical asymptote at $x = -3$

Oblique asymptote at $y = 2x - 1$



The graph may have stationary points. Solve $\frac{dy}{dx} = 0$

Make sure you include the x and y intercepts in sketches.

The Rational Function Case 3: $f(x) = \frac{ax+b}{cx^2+dx+e}, cx^2 + dx + e \neq 0$

The graph has

- vertical asymptote(s) where $cx^2 + dx + e = 0$
- a horizontal asymptote at $y = 0$

Check the numerator and the denominator for common factors

The graph may have stationary points. Solve $\frac{dy}{dx} = 0$

These graphs are a little less predictable, so it is useful to explore the behaviour of the function

- as $x \to \pm\infty$
- close to the vertical asymptotes

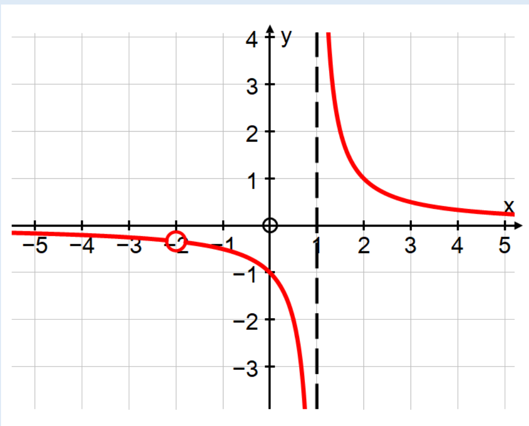
Make sure you include the x and y intercepts in sketches.

$$\mathrm{f}(x) = \frac{x+2}{x^2+x-2}$$

$$f(x) = \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$$

$$\mathrm{f(x)} = \frac{1}{x-1}\ ,\quad \mathrm{x} \neq 1$$

Vertical asymptote *at* $x = 1$



$$\mathrm{f}(x) = \frac{x+1}{x^2+x-2}$$

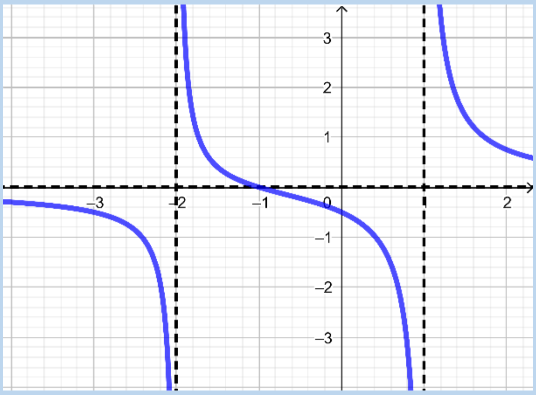
$$\mathrm{f(x)} = \frac{x+1}{(x+2)(x-1)},\ \mathrm{x} \neq -2\ ,\ \mathrm{x} \neq 1$$

Vertical asymptote at $x = -2, x = 1$

Horizontal asymptote at $y = 0$

x intercept where x = 1 = 0, x = -1

y intercept where $y = \frac{0+1}{(0+2)(0-1)}, y = -\frac{1}{2}$



Notice that even though there is an asymptote at $y = 0$, the graph can cross the x axis