

Find the value(s) of a such that $f(x) = \frac{x+1}{ax^2+3x+2}$ has only one vertical asymptote

If $f(x)$ has only one vertical asymptote there are 2 possibilities

- 1) $ax^2 + 3x + 2$ has 2 equal factors
- 2) $x + 1$ and $ax^2 + 3x + 2$ have a common factor

1)

$ax^2 + 3x + 2$ has 2 equal factors means that
 $ax^2 + 3x + 2 = 0$ has 1 repeated root

Therefore, $\Delta = 0$

Remember that $\Delta = b^2 - 4ac$

$$3^2 - 4a(2) = 0$$

$$9 - 8a = 0$$

$$a = \frac{9}{8}$$

2)

$$f(x) = \frac{x+1}{(x+1)(\dots)}$$

$$(x+1)(\dots) \equiv ax^2 + 3x + 2$$

$$(x+1)(x+2) \equiv ax^2 + 3x + 2$$

$$a = 1$$

Hence

$$a = 1, \frac{9}{8}$$