Sketch the graph of $f(x) = \frac{x^2+x-1}{x+2}$ giving the equations of any asymptotes and the coordinates of the x and y intercepts as well as any stationary points

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$f(x) = \frac{x^2+x-1}{x+2}$ has a vertical asymptote where $x + 2 = 0$

Vertical asymptote at $x =$ -2

Check for any common factors with numerator and denominator.

Let $g(x) = x^2 + x - 1$

$g(\text{-}2) = (\text{-}2)^2 + (\text{-}2) - 1 = 1$

Therefore, $x + 2$ is not a factor of $x^2 + x - 1$

Find the equation of the oblique asymptote

$$\begin{array}{r|l} & \quad\; x - 1 \\ x + 2 & x^2 + x - 1 \\ & \underline{x^2 + 2x} \\ & \quad\; -x - 1 \\ & \quad\; \underline{-x - 2} \\ & \qquad\quad 1 \end{array}$$

$$f(x) = x - 1 + \frac{1}{x+2}$$

Asymptote at $y = x - 1$

Find the y intercept

$$y = \frac{0^2 + 0 - 1}{0 + 2}$$

$$y = -\frac{1}{2}$$

y intercept at $\left(0, -\frac{1}{2}\right)$

Find the x intercepts

$\frac{x^2+x-1}{x+2} = 0, x \neq -2$

$x^2 + x - 1 = 0$

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$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

x intercepts at $\left(\frac{-1-\sqrt{5}}{2}, 0\right)$ and $\left(\frac{-1+\sqrt{5}}{2}, 0\right)$

Find any stationary points

Solve $f'(x) = 0$

$$f(x) = x - 1 + \frac{1}{x+2}$$

$$f'(x) = 1 - \frac{1}{(x+2)^2}$$

$$1 - \frac{1}{(x+2)^2} = 0$$

$$1 = \frac{1}{(x+2)^2}$$

$$(x+2)^2 = 1$$

$$x = -2 \pm 1$$

$$x = -3, -1$$

$$f(-3) = -3 - 1 + \frac{1}{-3+2} = -5$$

$$f(-1) = -1 - 1 + \frac{1}{-1+2} = -1$$

Stationary points at (-3, -5) and (-1, -1)

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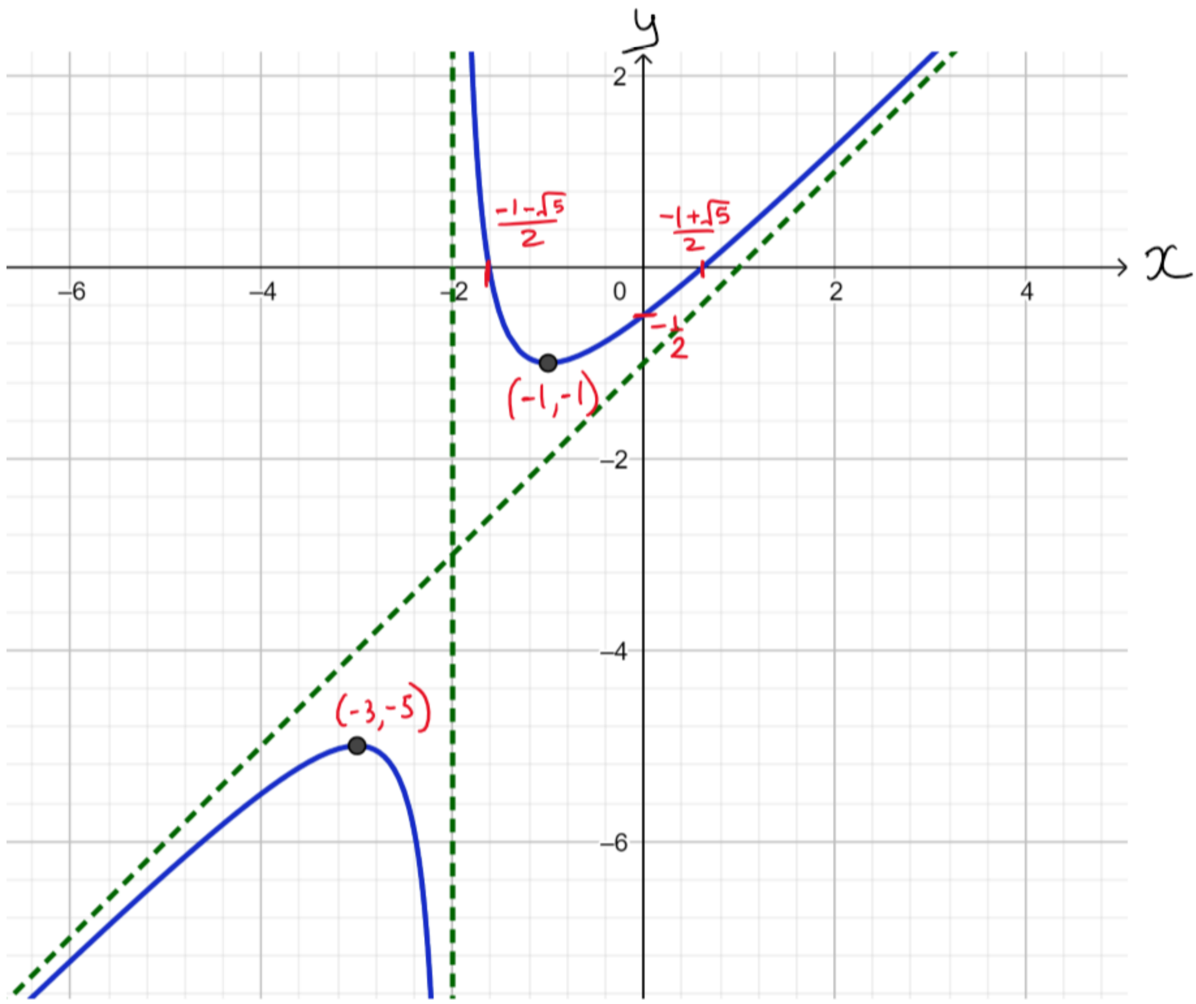

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