

Factor and Remainder Theorem

The remainder theorem states that for a polynomial $f(x)$,
the remainder when divided by $(x-a)$ is $f(a)$

The factor theorem states that for a polynomial $f(x)$,
 $(x-a)$ is a factor if and only if $f(a)=0$

The Factor Theorem is a theorem that allows us to find factors of polynomial functions, to find zeros and ultimately to help us sketch graphs.

In examinations, we may be required to factorise cubic functions (or other polynomials). We can use the factor theorem to find one or more factors, then we can use a method of inspection to find any further factors

e.g. Factorise the cubic function $f(x) = x^3 - 12x - 16$

$$f(-2) = (-2)^3 - 12(-2) - 16 = 0 \quad \text{therefore } (x + 2) \text{ is a factor}$$

$$f(4) = (4)^3 - 12(4) - 16 = 0 \quad \text{therefore } (x - 4) \text{ is a factor}$$

The factor theorem **does not reveal** any further factors...

We can work out the final factor by inspection

$$x^3 - 12x - 16 = (x + 2)(x - 4)(ax + b)$$

$$x^3 - 12x - 16 = (x^2 - 2x - 8)(ax + b)$$

You can work out the values of a and

$$x^3 - 12x - 16 = (x^2 - 2x - 8)(x + 2)$$

$$x^3 - 12x - 16 = (x + 2)(x - 4)(x + 2)$$