The remainder theorem states that for a polynomial f(x),

the remainder when divided by (x-a) is f(a)

The factor theorem states that for a polynomial f(x),

(x-a) is a factor if and only if f(a)=0

Factorize completely $f(x) = 2x^3 + x^2 - 7x - 6$

$$f(x) = 2x^3 + x^2 - 7x - 6$$

(x-a) is a factor if and only if f(a)=0

Linear factors in the form $(ax \pm b)$
$a \in \{1,2\}$
$b \in \{1,2,3,6\}$

$$f(1) = 2(1)^3 + (1)^2 - 7(1) - 6$$
$$f(1) = 2 + 1 - 7 - 6 \neq 0$$

$(x-1)$ is not a factor

$$f(-1) = 2(-1)^3 + (-1)^2 - 7(-1) - 6$$
$$f(-1) = -2 + 1 + 7 - 6 = 0$$

$(x+1)$ is a factor

$$f(2) = 2(2)^3 + (2)^2 - 7(2) - 6$$
$$f(2) = 16 + 4 - 14 - 6 = 0$$

$(x-2)$ is a factor

$$2x^3 + x^2 - 7x - 6 = (x+1)(x-2)(ax+b)$$

$$(x+1)(x-2) = x^2 - x - 2$$

$$2x^3 + x^2 - 7x - 6 = (x^2 - x - 2)(2x + 3)$$

$$f(x) = (x+1)(x-2)(2x+3)$$