The cubic polynomial $ax^3 + bx^2 - 29x + 60$ has a factor $(x + 4)$ and leaves a remainder of 6 when divided by $(x - 2)$.

a) Find the value of a and the value of b.
b) Factorize the polynomial

Let $f(x) = ax^3 + bx^2 - 29x + 60$

$(x + 4)$ is a factor
$f(-4) = 0$

$$f(-4) = a(-4)^3 + b(-4)^2 - 29(-4) + 60 = 0$$
$$a \times (-64) + b \times 16 - 29 \times (-4) + 60 = 0$$
$$-64a + 16b = -176$$

Remainder of 6 when divided by $(x - 2)$
$f(2) = 6$

$$f(2) = a(2)^3 + b(2)^2 - 29(2) + 60 = 6$$
$$8a + 4b = 4$$

$$(8a + 4b = 4) \times 4$$
$$-64a + 16b = -176$$
$$32a + 16b = 16$$
$$96a = 192$$
$$a = 2$$
$$b = -3$$

$$2x^3 - 3x^2 - 29x + 60 = (x + 4)(2x^2 + qx + r)$$
$$2x^3 - 3x^2 - 29x + 60 = (x + 4)(2x^2 + qx + 15)$$
$$2x^3 - 3x^2 - 29x + 60 = (x + 4)(2x^2 + qx + 15)$$

$$8x^2 + qx^2 = -3x^2$$
$$q = -11$$

$$2x^3 - 3x^2 - 29x + 60 = (x + 4)(2x^2 - 11x + 15)$$
$$= (x + 4)(2x - 5)(x - 3)$$