Let $f(x) = e^{2x}cosx$

a) Find $f'(x)$

b) Show that $f''(x) = 4f'(x) - 5f(x)$

a) $f(x) = e^{2x}cosx$

Use the Product Rule to find $f'(x)$

$f(x) = g(x)h(x)$

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$f'(x) = 2e^{2x}cosx + e^{2x}(-sinx)$

$f'(x) = 2e^{2x}cosx - e^{2x}sinx$

$f'(x) = e^{2x}(2cosx - sinx)$

$b)\ f'(x) = e^{2x}(2cosx - sinx)$

Use the product rule again

$f''(x) = 2e^{2x}(2cosx - sinx) + e^{2x}(-2sinx - cosx)$

$f''(x) = e^{2x}(4cosx - 2sinx) + e^{2x}(-2sinx - cosx)$

$f''(x) = e^{2x}(3cosx - 4sinx)$

$f''(x) = e^{2x}(8cosx - 4sinx) - e^{2x}(5cosx)$

$f''(x) = 4e^{2x}(2cosx - sinx) - 5e^{2x}(cosx)$

$f''(x) = 4f'(x) - 5f(x)$