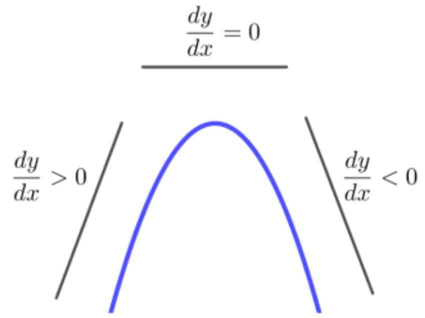
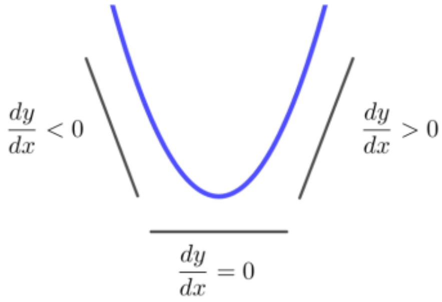
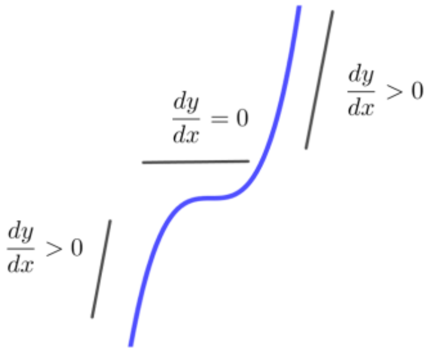
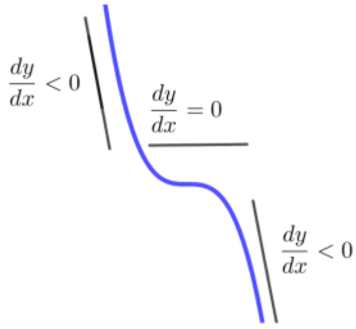
Find the co-ordinates of the stationary points on the curve
y = $x^4 - 4x^3$ and determine their nature. Sketch the curve.









$Solve\ \frac{dy}{dx} = 0$

- $if\ \frac{d^2y}{dx^2} < 0$ ...then local maximum
- $if\ \frac{d^2y}{dx^2} > 0$ ...then local minimum
- $if\ \frac{d^2y}{dx^2} = 0$ ...we cannot say Check $\frac{dy}{dx}$ before & after

$$y = x^4 - 4x^3$$

Differentiate with respect to x

$$\frac{dy}{dx} = 4x^3 - 4(3x^2)$$
$$\frac{dy}{dx} = 4x^3 - 12x^2$$

Stationary points occur where $\frac{dy}{dx} = 0$

Solve $\frac{dy}{dx} = 0$

$$4x^3 - 12x^2 = 0$$
$$4x^2(x - 3) = 0$$
$$4x^2 = 0\ ,\qquad x - 3 = 0$$

$x = 0$, $x = 3$

Find y coordinates

When $x = 0$, $y = (0)^4 - 4(0)^3 = 0$

When $x = 3$, $y = (3)^4 - 4(3)^3 = -27$

$$\frac{dy}{dx} = 4x^3 - 12x^2$$

Differentiate with respect to x

$$\frac{d^2y}{dx^2} = 12x^2 - 24x$$

Find the sign of $\frac{d^2y}{dx^2}$ for each stationary point

When $x = 3$,

$$\frac{d^2y}{dx^2} = 12(3)^2 - 24(3) = 36 > 0$$

Since $\frac{d^2y}{dx^2} > 0$

⇒Local Minimum at $x = 3$

Now check for $x = 0$

When $x = 0$,

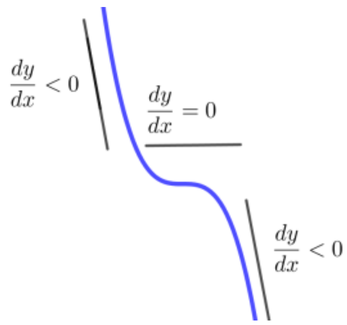
$$\frac{d^2y}{dx^2} = 12(0)^2 - 24(0) = 0$$

Need to check $\frac{dy}{dx}$ before and after

When $x = -1$,

$$\frac{dy}{dx} = 4(-1)^3 - 12(-1)^2 < 0$$

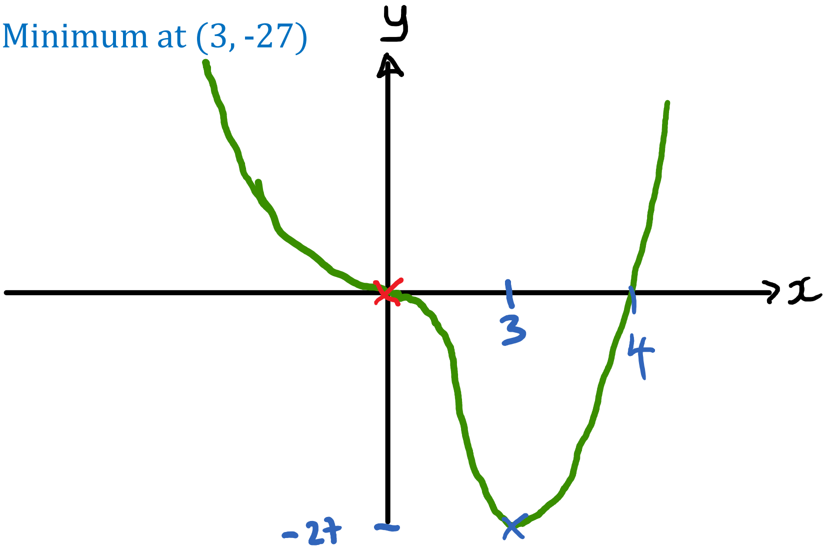
When $x = 1$, $\frac{dy}{dx} = 4(1)^3 - 12(1)^2 < 0$



⇒Point of inflexion at $x = 0$

Point of inflexion at (0,0)

Local Minimum at (3, -27)



$y = x^4 - 4x^3$

$x^4 - 4x^3 = 0$

$x^3(x - 4) = 0$

$x = 0\,, x = 4$