Proof by Induction

There are 5 steps in each proof by induction

1. Set up the proposition P(n) …
2. Show that proposition is true for n = 1
3. Assume that it is true for n = k
4. Show that it is true for n = k + 1
5. Write concluding statement

The key part of the proof is step 4 in which we use some information that we have assumed in step 3 to show that the proposition is true for n = k + 1. In this step, you may have to do some clever algebraic manipulation. Here is an example:

Assume true for $n = k$

$$1^3 + 2^3 + 3^3 + \cdots + k^3 \equiv \left(\frac{k(k+1)}{2}\right)^2$$

Show true for $n = k + 1$

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 \equiv \left(\frac{(k+1)(k+2)}{2}\right)^2$$

$$LHS \equiv 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3$$

$$\equiv \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

There are 3 straight-forward proofs that you may be required to carry out and you should learn how to do these by heart:

a) A divisibility test
b) A series
c) An inequality

Other more challenging proofs can involve trigonometry, complex numbers and differentiation. The method for these follow the same principals, but you may need to show a greater deal of creativity in your proofs. Ensure that you have any useful formulae in front of you so that you may recognise the steps that you need to make and get lots of practice!