Let $y = \frac{1}{1-x}$, $x \epsilon \mathbb{R}$

Prove by induction that $\frac{d^n y}{dx^n} = \frac{n!}{(1-x)^{n+1}}$, $n \in \mathcal{Z}^+$

Let P(n) be the proposition that $\frac{d^n y}{dx^n} = \frac{n!}{(1-x)^{n+1}}$, $n \in \mathcal{Z}^+$

Show true for $n = 1$

$$y = \frac{1}{1-x}$$

$$y = (1-x)^{-1}$$

$$\frac{dy}{dx} = (-1)(-1)(1-x)^{-2}$$

$$\frac{dy}{dx} = \frac{1}{(1-x)^2}$$

$$\frac{d^1 y}{dx^1} = \frac{1!}{(1-x)^{1+1}}$$

$$\frac{d^1 y}{dx^1} = \frac{1}{(1-x)^2}$$

Hence true for $n = 1$

Assume true for $n = k$

Assume that $\frac{d^k y}{dx^k} = \frac{k!}{(1-x)^{k+1}}$ is true

Show true for $n = k+1$

Show that $\frac{d^{k+1} y}{dx^{k+1}} = \frac{(k+1)!}{(1-x)^{k+2}}$ is true

$$\frac{d^{k+1} y}{dx^{k+1}} = \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right)$$

$$= \frac{d}{dx}\left(\frac{k!}{(1-x)^{k+1}}\right)$$

$$= k!\frac{d}{dx}((1-x)^{-k-1})$$

$$= k!\,(-k-1)(-1)(1-x)^{-k-2}$$

$$= k!\,(k+1)(1-x)^{-k-2}$$

$$= \frac{k!\,(k+1)}{(1-x)^{k+2}}$$

$$= \frac{(k+1)!}{(1-x)^{k+2}}$$

Hence true for $n = k + 1$

Concluding statement

True for $n = 1$
Assuming it is true for $n = k$ then it is true for $n = k + 1$
Therefore it is true for all $n \in \mathbb{Z}^+$