

## Proof by Induction

Prove that  $n^3 + 11n$  is divisible by 3 for  $n \in \mathbb{Z}, n > 0$

Show true for  $n = 1$

$$\begin{aligned}1^3 + 11 \times 1 &= 12 \\12 &= 3 \times 4\end{aligned}$$

Assume true for  $n = k$

Assume  $k^3 + 11k$  is divisible by 3

$$k^3 + 11k = 3m \quad m \in \mathbb{Z}$$

Show true for  $n = k + 1$

Show  $(k + 1)^3 + 11(k + 1)$  is divisible by 3

$$\begin{aligned}(k + 1)^3 + 11(k + 1) &\equiv k^3 + 3k^2 + 3k + 1 + 11k + 11 \\&\equiv k^3 + 11k + 3k^2 + 3k + 12 \\&\equiv 3m + 3(k^2 + k + 4)\end{aligned}$$

Concluding statement

True for  $n = 1$

Assuming it is true for  $n = k$  then it is true for  $n = k + 1$

Therefore it is true for all  $n \in \mathbb{Z}, n > 0$