Prove that $1^3+2^3+3^3+\cdots+n^3 \equiv \left(\frac{n(n+1)}{2}\right)^2 \quad , n\epsilon\mathbb{N}$

Let P(n) be the proposition that $1^3+2^3+3^3+\cdots+n^3 \equiv \left(\frac{n(n+1)}{2}\right)^2 \quad , n\epsilon\mathbb{N}$

Show true for $n=1$

$LHS = 1^3 = 1$

$RHS = \left(\frac{1(1+1)}{2}\right)^2$

$RHS = \left(\frac{2}{2}\right)^2 = 1$

$LHS = RHS$

Hence true for $n=1$

Assume true for $n=k$

$$1^3+2^3+3^3+\cdots+k^3 \equiv \left(\frac{k(k+1)}{2}\right)^2$$

Show true for $n=k+1$

$$1^3+2^3+3^3+\cdots+k^3+(k+1)^3 \equiv \left(\frac{(k+1)(k+2)}{2}\right)^2$$

$$LHS \equiv 1^3+2^3+3^3+\cdots+k^3+(k+1)^3$$

$$\equiv \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

$$\equiv \frac{(k(k+1))^2}{4} + (k+1)^3$$

$$\equiv \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4}$$

$$\equiv \frac{k^2(k+1)^2}{4} + \frac{4(k+1)(k+1)^2}{4}$$

$$\equiv \frac{(k+1)^2}{4}(k^2+4(k+1))$$

$$\equiv \frac{(k+1)^2}{4}(k^2+4k+4)$$

$$\equiv \frac{(k+1)^2}{4}(k+2)^2$$

$$\equiv \frac{(k+1)^2(k+2)^2}{4}$$

$$\equiv \frac{[(k+1)(k+2)]^2}{2^2}$$

$$\equiv \left(\frac{(k+1)(k+2)}{2}\right)^2$$

$$\equiv RHS$$

Hence true for $n=k+1$

Concluding statement

True for $n = 1$
Assuming it is true for $n = k$ then it is true for $n = k + 1$
Therefore it is true for all $n \in \mathbb{N}$