Prove by Induction that $12^n + 2 \times 5^{n-1}$ is divisible by 7 for $n \epsilon \mathbb{Z}^+$

Let P(n) be the proposition that $12^n + 2 \times 5^{n-1}$ is divisible by 7 for $n \epsilon \mathbb{Z}^+$

Show true for $n = 1$

$12^1 + 2 \times 5^{1-1} = 12 + 2 \times 1 = 14$

$14 = 7 \times 2$

14 is divisble by 7

Hence true for $n = 1$

Assume true for $n = k$

Assume that $12^k + 2 \times 5^{k-1}$ is divisible by 7

$12^k + 2 \times 5^{k-1} \equiv 7m \ , m \epsilon \mathbb{Z}$

$12^k \equiv 7m - 2 \times 5^{k-1}$

Show true for $n = k + 1$

Show that $12^{k+1} + 2 \times 5^k$ is divisible by 7

$$
\begin{aligned}
12^{k+1} + 2 \times 5^k &\equiv 12 \times 12^k + 2 \times 5^k \\
&\equiv 12(7m - 2 \times 5^{k-1}) + 2 \times 5^k \\
&\equiv 84m - 24 \times 5^{k-1} + 2 \times 5^k \\
&\equiv 84m - 24 \times 5^{k-1} + 2 \times 5 \times 5^{k-1} \\
&\equiv 84m - 24 \times 5^{k-1} + 10 \times 5^{k-1} \\
&\equiv 84m - 14 \times 5^{k-1} \\
&\equiv 7(12m - 2 \times 5^{k-1})
\end{aligned}
$$

$7(12m - 2 \times 5^{k-1})$ is divisible by 7 provided that $k \epsilon \mathbb{Z}, k \geq 1$

Hence true for $n = k + 1$

Concluding statement

True for $n = 1$
Assuming it is true for $n = k$ then it is true for $n = k + 1$
Therefore it is true for all $n \in \mathbb{Z}^+$