Prove by contradiction that the length of the hypotenuse of a right-angled triangle is less than the sum of the other two sides.

Let c be the length of the hypotenuse

…and a and b be the lengths of the other two sides

We need to prove that $c < a + b$

Assume the opposite

$c \geq a + b$

Square both sides

$c^2 \geq (a + b)^2$

$c^2 \geq a^2 + 2ab + b^2$

If this is a right-angled triangle, then $c^2 = a^2 + b^2$

$c^2 \geq c^2 + 2ab$

$0 \geq 2ab$

$ab \leq 0$

Since a and b are lengths, then they are positive.

This is a contradiction

Therefore, $c < a + b$

We have proved that the length of the hypotenuse of a right-angled triangle is less than the sum of the other two sides.