Prove that $\sqrt[n]{p}$ is irrational, given that p is prime

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Let $r = \sqrt[n]{p}$

Assume that the opposite is true

Assume that r is rational

r can be written as a fraction in its lowest terms

$r = \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0$

...a and b are coprime

...there is **no** common factor of a and b other than 1

$r^n = \frac{a^n}{b^n} = p$

$a^n = pb^n$

pb^n is divisible by p

a^n is divisible by p

Using the Fundamental Theorem of Arithmetic and since 5 is prime:

a is divisible by p

$a = pn$

$a^n = pb^n$

$(pn)^n = pb^n$

$p^n n^n = pb^n$

$p^{n-1} n^n = b^n$

b^n is divisible by p

b is divisible by p

If a and b are both divisible by p, then $\frac{a}{b}$ has a common factor of p

a and b are NOT coprime

This is a contradiction

$\sqrt[n]{p}$ is not rational. We have proved that $\sqrt[n]{p}$ is irrational.