Prove that $\sqrt{3}$ is irrational

Let $r = \sqrt{3}$

Assume that the opposite is true

Assume that r is rational

r can be written as a fraction in its lowest terms

$r = \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0$

…a and b are coprime

…there is **no** common factor of a and b other than 1

$r^2 = \frac{a^2}{b^2} = 3$

$a^2 = 3b^2$

$3b^2$ is divisible by 3

a^2 is divisible by 3

Using the Fundamental Theorem of Arithmetic and since 3 is prime:

a is divisible by 3

$a = 3n$

$a^2 = 3b^2$

$(3n)^2 = 3b^2$

$9n^2 = 3b^2$

$3n^2 = b^2$

b^2 is divisible by 3

b is divisible by 3

If a and b are both divisible by 3, then $\frac{a}{b}$ has a common factor of 3

a and b are NOT coprime

This is a contradiction

$\sqrt{3}$ is not rational. We have proved that $\sqrt{3}$ is irrational.