Prove by contradiction that a rational number + an irrational number = irrational number.

Remember the definition of a rational and irrational numbers

- Rational numbers can be expressed as fractions.
- Irrational numbers cannot be expressed as fractions.

We can re-write the statement:

If a is a rational number and b is an irrational number, then $a + b$ is an irrational number

If a is a ***rational*** number then

$$a = \frac{m}{n},\ m, n \in \mathbb{Z}$$

Assume that the **opposite is true**, that is, that $a + b$ is a ***rational number***

If $a + b$ is a rational number then

$$a + b = \frac{p}{q}, p, q \in \mathbb{Z}$$

Write out the given statement

$$\frac{m}{n} + b = \frac{p}{q}$$

Rearrange to make b

$$b = \frac{p}{q} - \frac{m}{n}$$

$$b = \frac{np - mq}{qn}$$

$np - mq$ is an integer

qn is an integer

Therefore, $\frac{np-mq}{qn}$ is a rational number

That is b is a rational number

This is a **contradiction**. We know that b is an irrational number.

Therefore,

a rational number + an irrational number = irrational number