a) Verify that ${}^{3}C_1 + {}^{3}C_2 = {}^{4}C_2$

b) Prove that ${}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^{n}C_r$

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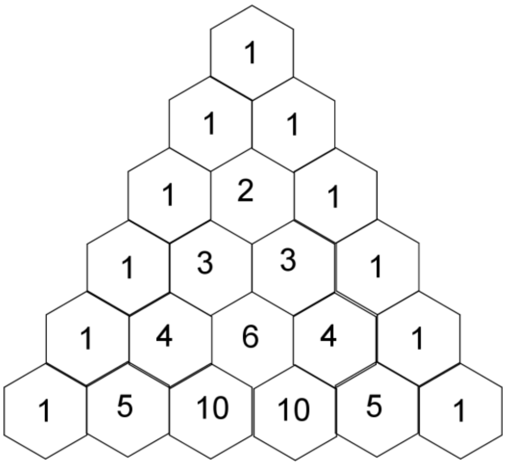
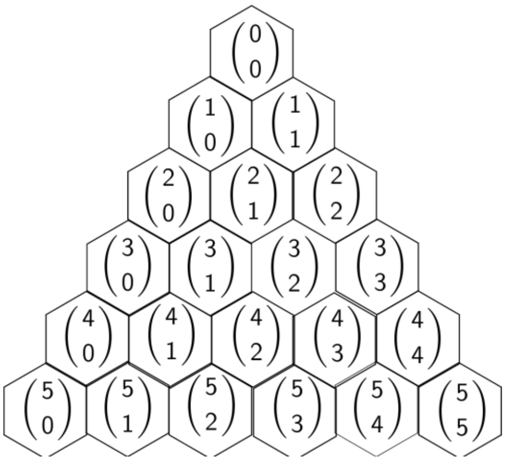
a)

$$\begin{aligned} {}^{3}C_1 + {}^{3}C_2 &= 3 + 3 \\ &= 6 \\ {}^{4}C_2 &= 6 \end{aligned}$$

We can see that this is the 4th row of Pascal's triangle





b) Prove that ${}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^{n}C_r$

$$\begin{aligned} LHS &= {}^{n-1}C_{r-1} + {}^{n-1}C_r \\ &= \frac{(n-1)!}{[(n-1)-(r-1)]!\,(r-1)!} + \frac{(n-1)!}{(n-1-r)!\,r!} \\ &= \frac{(n-1)!}{(n-r)!\,(r-1)!} + \frac{(n-1)!}{(n-r-1)!\,r!} \end{aligned}$$

To be able to simplify these algebraic fractions, we need to make the denominators the same. We need to be able to manipulate the factorials in this expression

$$r! = r \cdot (r-1)!$$

$$\begin{aligned} LHS &= \frac{r(n-1)!}{(n-r)!\,r(r-1)!} + \frac{(n-1)!}{(n-r-1)!\,r!} \\ &= \frac{r(n-1)!}{(n-r)!\,r!} + \frac{(n-1)!}{(n-r-1)!\,r!} \end{aligned}$$

$$= \frac{r(n-1)!}{(n-r)!\,r!} + \frac{(n-1)!}{(n-r-1)!\,r!}$$

Also,

$$(n-r)! = (n-r)(n-r-1)!$$

$$LHS = \frac{r(n-1)!}{(n-r)!\,r!} + \frac{(n-r)(n-1)!}{(n-r)(n-r-1)!\,r!}$$

$$= \frac{r(n-1)!}{(n-r)!\,r!} + \frac{(n-r)(n-1)!}{(n-r)!\,r!}$$

$$= \frac{r(n-1)! + (n-r)(n-1)!}{(n-r)!\,r!}$$

$$= \frac{r(n-1)! + n(n-1)! - r(n-1)!}{(n-r)!\,r!}$$

$$= \frac{n(n-1)!}{(n-r)!\,r!}$$

$$n! = n(n-1)!$$

$$LHS = \frac{n!}{(n-r)!\,r!}$$

$$= {}^{n}C_{r}$$

$$= RHS$$

Notice that what we have proved is that, if you take any two consecutive terms from Pascal's triangle, then they add up to give the term below