

a) Verify that  $\frac{1}{x^2} - \frac{1}{(x+1)^2} = \frac{2x+1}{x^2(x+1)^2}$  for  $x = -2$

b) Prove that  $\frac{1}{x^2} - \frac{1}{(x+1)^2} \equiv \frac{2x+1}{x^2(x+1)^2}$  for all  $x$

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a) 
$$LHS = \frac{1}{x^2} - \frac{1}{(x+1)^2}$$

When  $x = -2$ ,  $LHS = \frac{1}{(-2)^2} - \frac{1}{(-2+1)^2}$

$$= \frac{1}{4} - \frac{1}{1}$$
$$= -\frac{3}{4}$$

$$RHS = \frac{2x+1}{x^2(x+1)^2}$$

When  $x = -2$ ,  $RHS = \frac{2(-2)+1}{(-2)^2(-2+1)^2}$

$$= \frac{-3}{4 \cdot 1}$$
$$= -\frac{3}{4}$$

b) 
$$LHS \equiv \frac{1}{x^2} - \frac{1}{(x+1)^2}$$

$$\equiv \frac{(x+1)^2}{x^2(x+1)^2} - \frac{x^2}{x^2(x+1)^2}$$

$$\equiv \frac{x^2 + 2x + 1}{x^2(x+1)^2} - \frac{x^2}{x^2(x+1)^2}$$

$$\equiv \frac{2x+1}{x^2(x+1)^2}$$

$$\equiv RHS$$