

$2 - 3i$ is a root to the equation $z^3 - 7z^2 + az + b = 0$, $a, b \in \mathbb{R}$

Work out a and b and the other roots of the equation.

<p>Since this is a cubic equation and we know that it has a complex root, therefore we know</p> <ul style="list-style-type: none"> it has 2 complex roots and 1 real root 	
	<p>If $z = 2 - 3i$ is a root to the polynomial</p> <p>Then $z = 2 + 3i$ is another root</p>
	<p>$(z - 2 + 3i)$ is a factor $(z - 2 - 3i)$ is a factor</p>
<p>Multiply these two together to find a quadratic factor</p>	
$(z - 2 + 3i)(z - 2 - 3i)$	$= z(z - 2 - 3i)$ $-2(z - 2 - 3i)$ $+3i(z - 2 - 3i)$
	$= z^2 - 2z - 3zi$ $-2z + 4 + 6i$ $+3zi - 6i - 9i^2$
	$= z^2 - 4z + 13$
<p>There must be 1 real factor. Call it $z - p$</p>	
$(z - p)(z^2 - 4z + 13)$	$= z^3 - 7z^2 + az + b$
<p>We can work out p, by considering how we get $-7z^2$</p>	$z(-4z) - p(z^2) = -7z^2$ $-4z^2 - pz^2 = -7z^2$ $p = 3$
<p>We now know the 3 factors $(z - 3)(z^2 - 4z + 13)$</p>	
$(z - 3)(z^2 - 4z + 13)$	$= z(z^2 - 4z + 13)$ $-3(z^2 - 4z + 13)$ $= z^3 - 4z + 13z$ $-3z^2 + 12z - 39$ $= z^3 - 7z^2 + 25z - 39$
<p>$a = 25$, $b = -39$</p> <p>Roots are $z = 3$, $2 - 3i$, $2 + 3i$</p>	



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