The equation $2z^4 - 9z^3 + pz^2 + qz - 174 = 0$, $p, q \in \mathbb{Z}$ has two real roots α and β and two complex roots γ and δ, where $\gamma = 2 - 5i$.

a. Show that $\alpha + \beta = \frac{1}{2}$.
b. Find $\alpha\beta$.
c. **Hence** find the two real roots α and β.
d. Find the values of p and q.

a.

$$2z^4 - 9z^3 + pz^2 + qz - 174 = 0$$

If $\gamma = 2 - 5i$ is a root ... then $\delta = 2 + 5i$ is also a root

Sum of roots $= \frac{9}{2}$

$$\alpha + \beta + \gamma + \delta = \frac{9}{2}$$

$$\alpha + \beta + 2 - 5i + 2 + 5i = \frac{9}{2}$$

$$\alpha + \beta + 4 = \frac{9}{2}$$

$$\alpha + \beta = \frac{1}{2}$$

b.

Product of roots $= -\frac{174}{2} = -87$

$$\alpha\beta\gamma\delta = -87$$

$$\alpha\beta(2 - 5i)(2 + 5i) = -87$$

$$\alpha\beta(4 - 25i^2) = -87$$

$i^2 = -1$

$$\alpha\beta(29) = -87$$

$$\alpha\beta = -3$$

c.

$$\alpha + \beta = \frac{1}{2} \Rightarrow \beta = \frac{1}{2} - \alpha$$

$$\alpha\beta = -3$$

$$\alpha\left(\frac{1}{2} - \alpha\right) = -3$$

$$\frac{1}{2}\alpha - \alpha^2 = -3$$

$$\alpha - 2\alpha^2 = -6$$

$$2\alpha^2 - \alpha - 6 = 0$$

$$(2\alpha + 3)(\alpha - 2) = 0$$

$$\alpha = -\frac{3}{2}, \alpha = 2$$

$$\beta = 2, \beta = -\frac{3}{2}$$

The two real roots are $2, -\frac{3}{2}$

d.

The equation is $a(z - 2)(2z + 3)\big(z - (2 - 5i)\big)\big(z - (2 + 5i)\big) = 0$

Since $2z^4 - 9z^3 + pz^2 + qz - 174 = 0$

...then $a = 1$

$$(z-2)(2z+3)\big(z-(2-5i)\big)\big(z-(2+5i)\big) = 0$$

$$(2z^2 - z - 6)\big(z^2 - (2+5i)z - (2-5i)z + (2+5i)(2-5i)\big) = 0$$

$$(2z^2 - z - 6)(z^2 - 4z + 4 - 25i^2) = 0$$

$$(2z^2 - z - 6)(z^2 - 4z + 29) = 0$$

$$2z^4 - 9z^3 + pz^2 + qz - 174 \equiv (2z^2 - z - 6)(z^2 - 4z + 29)$$

$p = 56$

$$2z^4 - 9z^3 + pz^2 + qz - 174 \equiv (2z^2 - z - 6)(z^2 - 4z + 29)$$

$q = -5$