

One root of the equation $z^2 + bz + c = 0$ is $2 + 3i$ where $b, c \in \mathbb{Z}$.
Find the value of b and the value of c .

If $z = 2 + 3i$ is a root ...then, the complex conjugate $z^* = 2 - 3i$ is also a root

Factors of the equation are $z - (2 + 3i)$
 $z - (2 - 3i)$

The equation is $a(z - (2 + 3i))(z - (2 - 3i)) = 0$

$$a(z - 2 - 3i)(z - 2 + 3i) = 0$$

$$a(z^2 + (-2 + 3i)z + (-2 - 3i)z + (-2 - 3i)(-2 + 3i)) = 0$$

Since $z^2 + bz + c = 0$, $a = 1$

$$z^2 + (-2 + 3i)z + (-2 - 3i)z + (-2 - 3i)(-2 + 3i) = 0$$

$$z^2 - 4z + 4 - 9i^2 = 0$$

$$i^2 = -1$$

$$z^2 - 4z + 13 = 0$$