De Moivre's Theorem

Polar Form	$[r(cos\theta + isin\theta)]^n = r^n(\cos n\theta + isin\, n\theta)$
CIS Form	$(rcis)^n = r^n cisn\theta$
Euler Form	$(re^{i\theta})^n = r^n e^{in\theta}$

You are often required to convert between Cartesian and Polar Form, so draw sketches of Argand diagrams. Two examples are shown below.

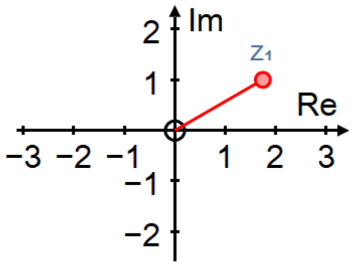
Powers of Complex Numbers

Given that $z = \sqrt{3} + i$, work out z^8

Write z in Polar Form.

$$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$



$$z = 2cis\frac{\pi}{6}$$

$$z^8 = \left(2cis\frac{\pi}{6}\right)^8$$

Use de Moivre's Theorem to find z^8

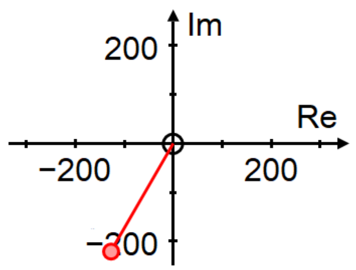
$$z^8 = 2^8 cis\left(8 \cdot \frac{\pi}{6}\right)$$

$$z^8 = 256cis\left(\frac{4\pi}{3}\right)$$

$$z^8 = 256cis\left(-\frac{2\pi}{3}\right)$$

We are often required to put the answer back into Cartesian Form

$$z^8 = 256\left(cos\left(-\frac{2\pi}{3}\right) + isin\left(-\frac{2\pi}{3}\right)\right)$$

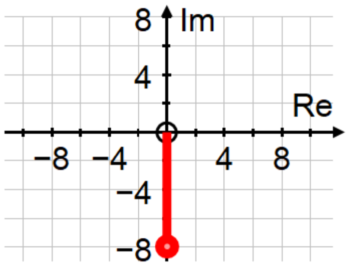
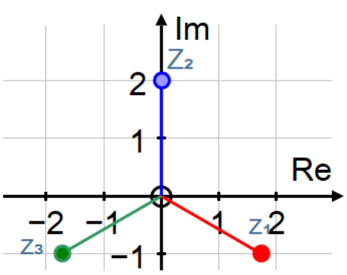


Notice that there is only **one answer**

$$z^8 = -128 - 128\sqrt{3}$$

Roots of Complex Numbers

Find the roots of the equation $z^3 = -8i$

Write z in Polar Form. 	$z^3 = -8i$
We give the possibility of finding 3 roots by adding $2k\pi$ for $k = 0,1,2$ to the argument	$z^3 = 8cis\left(-\frac{\pi}{2} + 2k\pi\right),\ k = 0,1,2$
Use de Moivre's Theorem...	$z = 8^{\frac{1}{3}}cis\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right),\ k = 0,1,2$
	$z = 2cis\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right),\ k = 0,1,2$
	$z_1 = 2cis\left(-\frac{\pi}{6}\right)$ $z_2 = 2cis\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right)$ $z_3 = 2cis\left(-\frac{\pi}{6} + \frac{4\pi}{3}\right)$
Using the symmetry of the roots helps to convert into Cartesian Form	$z_1 = 2cis\left(-\frac{\pi}{6}\right)$ $z_2 = 2cis\frac{\pi}{2}$ $z_3 = 2cis\left(-\frac{5\pi}{6}\right)$
	$z_1 = \sqrt{3} - i$ $z_2 = 2i$ $z_3 = -\sqrt{3} - i$ Notice that the roots are arranged symmetrically around the Argand diagram